# Training Guide

# DOE++

Version 1

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**DOE++ 1 Training Guide**
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# DOE++ Training Guide 1

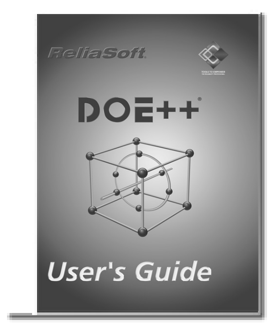
## 1.1 About This Training Guide

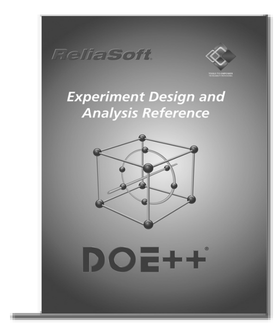
This training guide is intended to provide you with many examples of the DOE++ software. It begins with step-by-step examples and then proceeds into more advanced examples and questions. At any time during the training, please feel free to ask the instructor(s) any questions you might have.

If you have a demonstration version of DOE++, you may not be able to save your work for these examples. For this reason, you may wish to work with the sample projects that are shipped with the software instead of creating your own. In addition, the data sets for the examples are provided in Spreadsheets within separate files. Each of these data sets is sorted according to the standard order as specified in the relevant design type, making it easy for you to copy the data set and paste it into the Folio(s) that you create while working through the example. The names of the complete sample project and the data file that are shipped with the application are given for each example in this training guide.

## 1.2 DOE++ Documentation

Like all of ReliaSoft's standard software products, DOE++ is shipped with detailed printed documentation on the product (*DOE++ User's Guide*) and the underlying principles and theory (*ReliaSoft's Experiment Design and Analysis Reference*). This training guide is intended to be a supplement to those references.

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# Features Summary 2

The examples included in this training guide have been designed to introduce you to the features available in DOE++. This section presents a brief summary of these features. If you are already familiar with DOE++'s features, you can proceed to the examples.

## 2.1 Design of Experiments (DOE) Software... Designed With Reliability in Mind

ReliaSoft's DOE++ facilitates traditional Design of Experiments (DOE) techniques for studying the factors that may affect a product or process in order to identify significant factors and optimize designs. The software also expands upon standard methods to provide the proper analysis treatment for interval and right censored data — offering a major breakthrough for reliability-related analyses!

DOE++ guides you through the designs and analyses necessary for all phases of the DOE strategy, from screening for significant factors, through in-depth analysis of the targeted factors and factorial interactions, to selecting input levels for optimal performance.

## 2.2 Intuitive and Flexible Work Environment

The powerful and flexible interface allows you to create multiple designs in a single project, keeping all related analyses and information together in a single file. Using the same "Project Explorer" approach that is employed in ReliaSoft's Weibull++, ALTA and BlockSim software, DOE++ provides an intuitive, hierarchical (tree) structure to allow you to view and manage one or many standard folios, plot sheets, free-form regression analyses, spreadsheets and/or attached documents per project.

If you have Weibull++ and/or ALTA 7.5 or later installed on your computer, the applications will work in concert. This means that you can include all of the analysis types available in DOE++, Weibull++ and ALTA in a single project and switch between applications with the click of a button!

## 2.3 Supported Design Types

DOE++ facilitates the design types employed in traditional DOE and also expands upon the traditional methods to support "Reliability DOE," which provides the proper treatment for interval and right censored data. Supported design types include:

- **One Factor Designs**
- **Factorial Designs**
    - Two Level Full Factorial
    - Two Level Fractional Factorial
    - Plackett-Burman

  - General Full Factorial
  - Taguchi Orthogonal Array (single level or mixed level)
- **Response Surface Method Designs**
  - Central Composite
  - Box-Behnken
- **Taguchi Robust Designs**
- **Reliability DOE**
  - One Factor Reliability Designs
  - Two Level Full Factorial Reliability Designs
  - Two Level Fractional Factorial Reliability Designs
  - Plackett-Burman Reliability Designs

In addition, the software's Multiple Linear Regression Tool allows you to enter your own free-form factor and response data and perform similar analyses.

## 2.4 Detailed Analysis of Experimental Results

DOE++ provides a flexible array of tools for detailed analysis of experimental results, including:

- Analysis of variance (ANOVA) information - an overview of the effects of factor(s) on response(s).
- Level-specific information and comparisons for one factor designs.
- Breakdown of the importance of individual factors, interactions and/or groups of effects to output.
- Likelihood ratio test results for reliability DOEs.

## 2.5 Extensive Plotting Capabilities

DOE++ offers a wide variety of diagnostic, interpretive and predictive plots. In addition to a selection of individual plots (which are available for each design type based on their relevance and utility), the flexible Side-by-Side Plots utility allows you to view multiple plots for a given response simultaneously and MultiPlots facilitate comparison of separate response analyses. Plots include:

- **Level Plots:**
  - Response vs. Level
  - Level Mean
  - Life Characteristic
  - Box Plot
  - Mean PDFs
  - Comparison Chart
- **Effect Plots:**
  - Effect Probability
  - Scatter Plot
  - Pareto Chart
  - Main Effects

- Interactions
- Interaction Matrix
- Cube Plot

- **Residual Plots:**
  - Residual Probability
  - Residual vs. Fitted
  - Residual vs. Run
  - Residual vs. Factor
  - Residual Histogram
  - Fitted vs. Actual
- **Diagnostic Plots:**
  - Leverage
  - Cook's Distance
  - Box-Cox Transformation

## 2.6 Put Your Findings Into Practice with Powerful Optimization Utility

You can specify the goal for each response in your analysis (as a maximum, minimum or target value). DOE++ will search for the combinations of factor settings that most effectively lead to the desired output and rank them according to their desirability. If certain responses are more important to optimize than others, or if the target is more narrowly defined, the software can take that into account. Solutions can be viewed graphically or numerically.

## 2.7 Configure the Workspace to Meet Individual Needs

DOE++ makes it easy to configure the interface and analysis settings to meet your specific preferences and needs. For example, the User Setup allows you to specify default options for data sheets, analysis settings, fonts/symbols, etc. The Plot Setup allows you to configure the appearance of the plots that are automatically generated by the software. In addition, you can customize the toolbars and/or adjust the appearance of the workspace by hiding/displaying or changing the position of the Project Explorer and other panels.

## 2.8 System Requirements

DOE++ is compiled and designed for Windows NT, 2000, XP and Vista and takes advantage of the features available in these platforms. Minimum system requirements: A 433-MHz Intel Pentium-class processor or equivalent, with 64MB RAM (128MB or more is recommended), SVGA display and at least 130 MB of hard disk space.

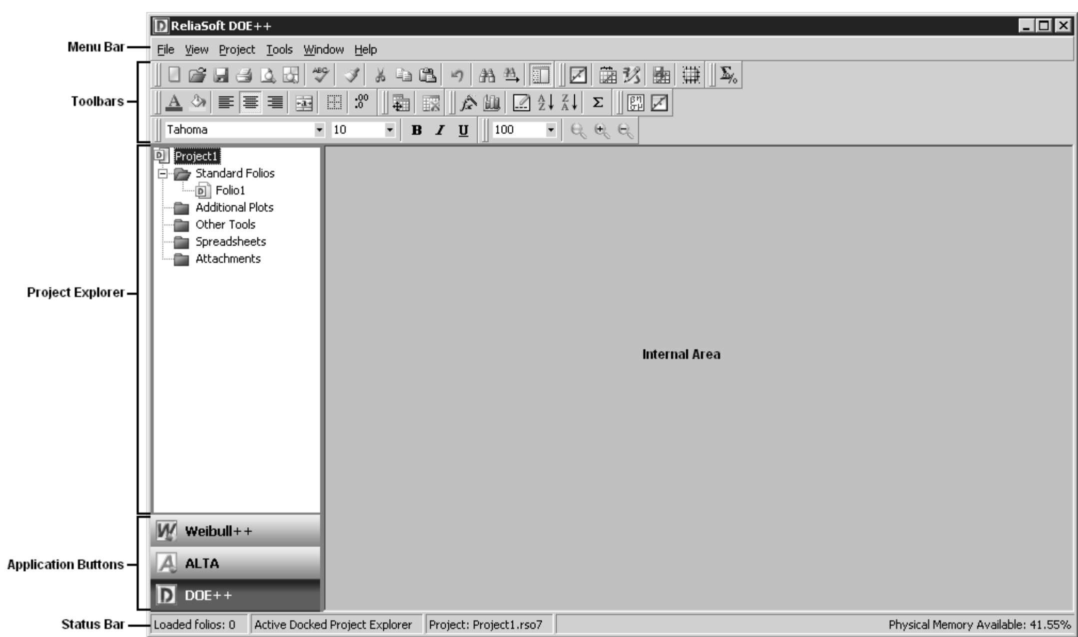
# First Steps 3

## 3.1 Starting DOE++

DOE++ is a 32-bit application that has been designed to work with Windows NT, 2000, XP and Vista. The internal screens and commands are identical regardless of which operating system you are using, and this training guide is equally applicable. To start DOE++, from **Start** select **Programs**, **ReliaSoft Office** and then **DOE++**.

## 3.2 Multiple Document Interface and Standard Folio

The Multiple Document Interface (MDI) serves as the container for the Project Explorer, all Standard Folios, Additional Plots, Other Tools (*i.e*. Multiple Linear Regression Tools), Spreadsheets and Attachments and manages the different active windows. The MDI remains open until you close the program. Closing the MDI has the same effect as terminating the program. The next figure shows the MDI of DOE++ and its components. The appearance of the MDI will vary depending on the window(s) that are currently open and the configuration settings of the Project Explorer.

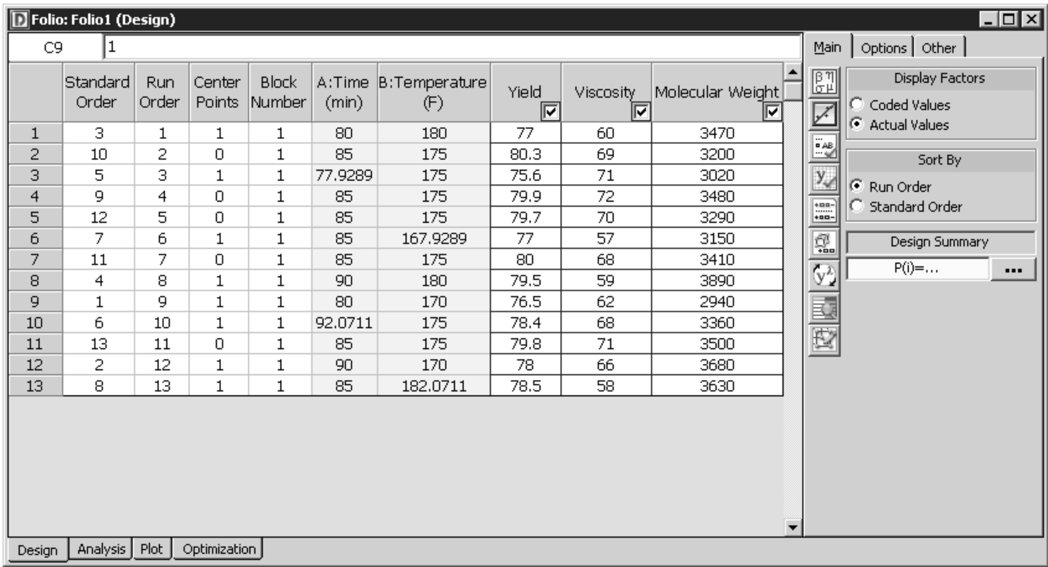



The Standard Folio allows you to enter and analyze the experimental data. It can consist of up to four tabs:

- The Design tab contains information pertaining to the design of the experiment and allows you to enter response data. This is the only tab present when the Folio is first created.
- The Analysis tab is created when you click **Calculate** in the Design tab and contains tables of calculated analytical information about the experiment.

- The Plot tab is created when you click **Plot** on the Main page of the Control Panel on either the Design tab or the Analysis tab and contains plots of the experiment analysis.
- The Optimization tab is created when you perform optimization (accessed from the Main page of the Analysis tab Control Panel) and contains a plot that graphically displays the settings for each factor that will yield the optimal response(s), as specified by the user.

The appearance of the Standard Folio varies depending on the type of design you are working with. A Standard Folio for a central composite design is shown in the figure below.




| | Standard Order | Run Order | Center Points | Block Number | A:Time (min) | B:Temperature (F) | Yield | Viscosity | Molecular Weight |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 1 | 1 | 1 | 80 | 180 | 77 | 60 | 3470 |
| 2 | 10 | 2 | 0 | 1 | 85 | 175 | 80.3 | 69 | 3200 |
| 3 | 5 | 3 | 1 | 1 | 77.9289 | 175 | 75.6 | 71 | 3020 |
| 4 | 9 | 4 | 0 | 1 | 85 | 175 | 79.9 | 72 | 3480 |
| 5 | 12 | 5 | 0 | 1 | 85 | 175 | 79.7 | 70 | 3290 |
| 6 | 7 | 6 | 1 | 1 | 85 | 167.9289 | 77 | 57 | 3150 |
| 7 | 11 | 7 | 0 | 1 | 85 | 175 | 80 | 68 | 3410 |
| 8 | 4 | 8 | 1 | 1 | 90 | 180 | 79.5 | 59 | 3890 |
| 9 | 1 | 9 | 1 | 1 | 80 | 170 | 76.5 | 62 | 2940 |
| 10 | 6 | 10 | 1 | 1 | 92.0711 | 175 | 78.4 | 68 | 3360 |
| 11 | 13 | 11 | 0 | 1 | 85 | 175 | 79.8 | 71 | 3500 |
| 12 | 2 | 12 | 1 | 1 | 90 | 170 | 78 | 66 | 3680 |
| 13 | 8 | 13 | 1 | 1 | 85 | 182.0711 | 78.5 | 58 | 3630 |

## 3.3 Getting Help in the DOE++ Environment

ReliaSoft's DOE++ includes complete on-line help documentation. This help can be obtained at any time by pressing **F1** or by selecting **Contents** from the **Help** menu. DOE++ also provides theory help, which is available for the signal-to-noise ratio options available for Taguchi robust designs and for each life distribution available for reliability designs. Theory help can be accessed by clicking the theory help button (**i**) next to your selection.

## 3.4 A Quick Overview Example

This section presents you with a very simple example and guides you through the solution.

While performing a failure modes and effects analysis (FMEA) of an engine, engineers identify the crankshaft as a critical area. Bending stress and speed are identified as two probable causes for fatigue failure. The engineers have identified the bending stress as a Key Process Input Variable (KPIV) and want to explore its effect. They also wish to explore the speed as a cause for fatigue failure and determine whether it is a significant factor. Finally, they would like to explore whether there is an important interaction between speed and bending stress.

The engineers decide on a $2^2$ factorial design with 3 replicates. The levels for the bending stress are chosen to be within design limits as 40 and 60 ksi. The speed levels, measured in terms of engine revolutions per minute, are chosen as 6 and 9 krpm. Because of time constraints, the response is measured in terms of degradation (crack size at a notch location) rather than times-to-failure. All units are tested to the same number of cycles.

The results obtained at each combination of factor settings are shown next.

| Factor A: Bending Stress (ksi) | Factor B: Speed (krpm) | Response: Crack Size (mm) |
|---|---|---|
| 40 | 6 | 0.33 |
| 60 | 6 | 0.38 |
| 40 | 9 | 0.19 |
| 60 | 9 | 0.47 |
| 40 | 6 | 0.48 |
| 60 | 6 | 0.74 |
| 40 | 9 | 0.3 |
| 60 | 9 | 0.77 |
| 40 | 6 | 0.2 |
| 60 | 6 | 0.5 |
| 40 | 9 | 0.3 |
| 60 | 9 | 0.54 |

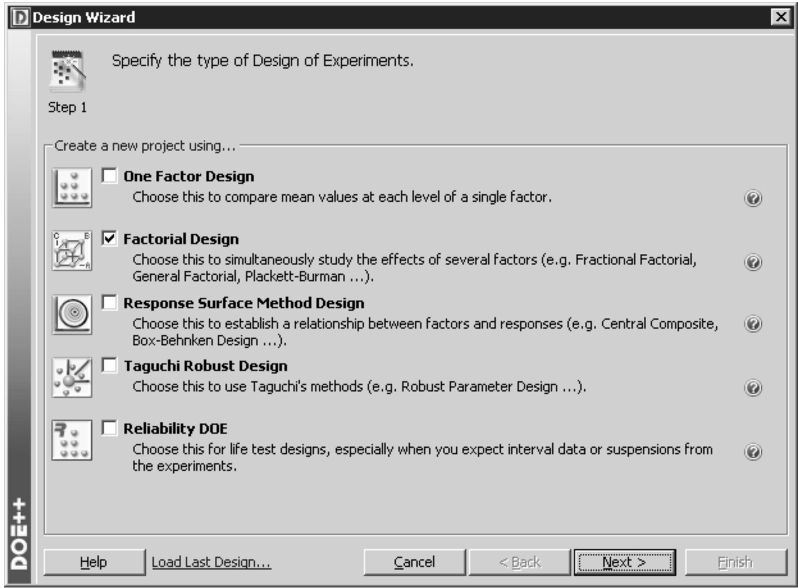
*The sample file for this example is located in the "Training Guide" folder in your application directory (e.g. C:\Program Files\ReliaSoft\DOE\Training Guide) and is named "1stSteps.rdoe." The data set for this example is available in the Spreadsheet in the "1stSteps-Data.rso7" file located in the "Raw Data" folder within the "Training Guide" folder.*

### 3.4.1 Creating the Project and Design

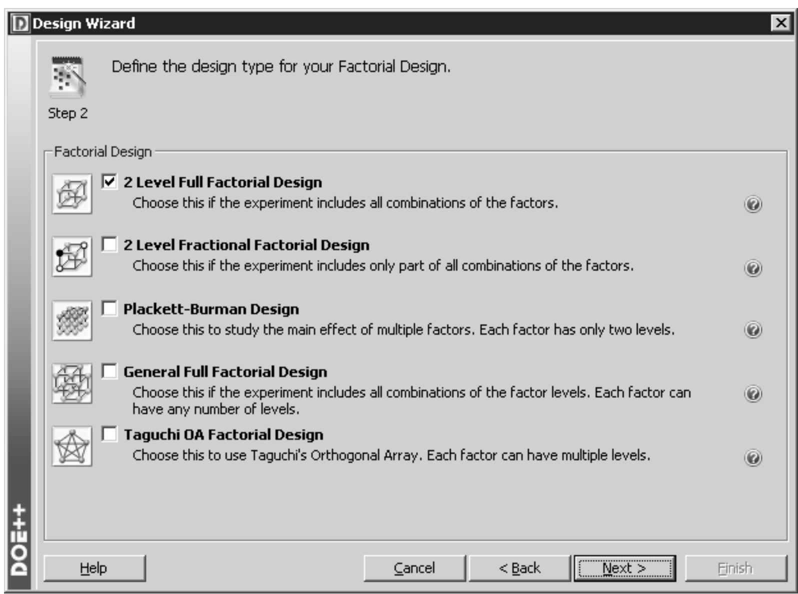
- Create a new project by clicking **Create a New Project** in the initial window that may appear at startup, by selecting **New** from the **File** menu or by clicking the **New** icon.

The Design Wizard will be displayed. This window consists of a series of pages that guide you through the process of creating a new project by asking you a series of questions that allow you to define the design used in the first Standard Folio that will be added to the project. The steps in the Design Wizard vary depending on the type of design you want to create. Note that you can click the **Load Last Design** link on the first page of the wizard to create another design with the same settings as the last design you created.

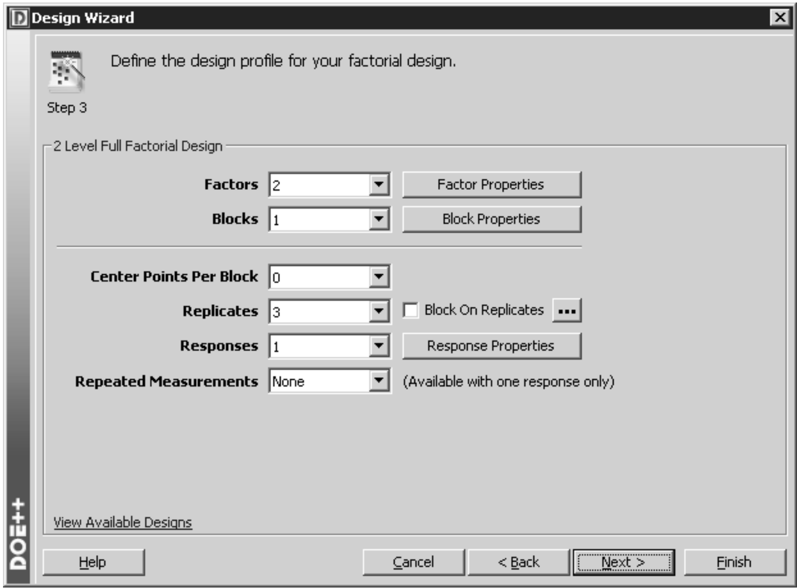
- On the first page of the Design Wizard, select to create a **Factorial Design** and click **Next** to proceed to the next step.



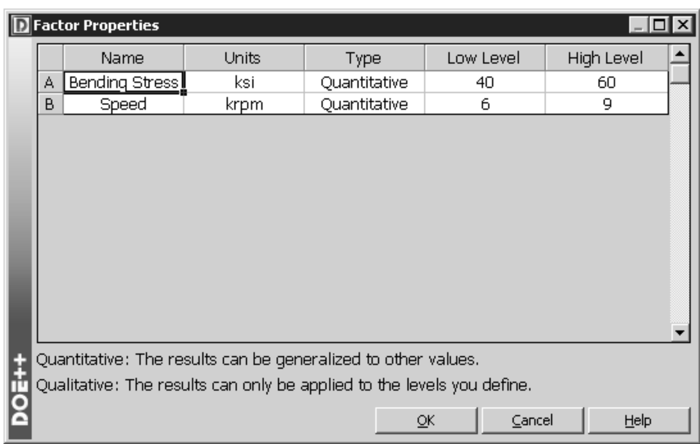

- On the second page of the Design Wizard, select **2 Level Full Factorial Design** and click **Next** to proceed to the next step.



- The third step of the Design Wizard allows you to define the exact configuration of the design. Select **2** from the **Factors** drop-down and **3** from the **Replicates** drop-down. Leave the other options set to the defaults, as shown next.



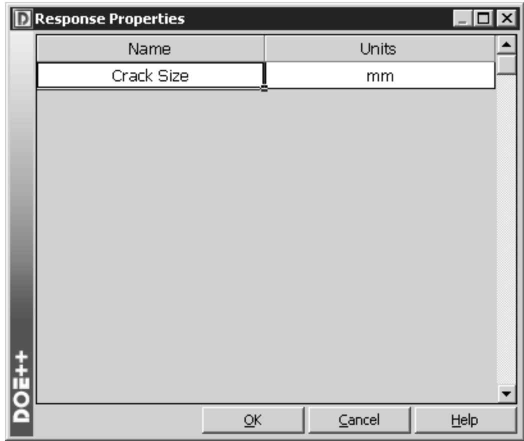

- Click the **Factor Properties** button. In the Factor Properties window that appears, enter the names, units and high and low level values for the factors, as shown next.



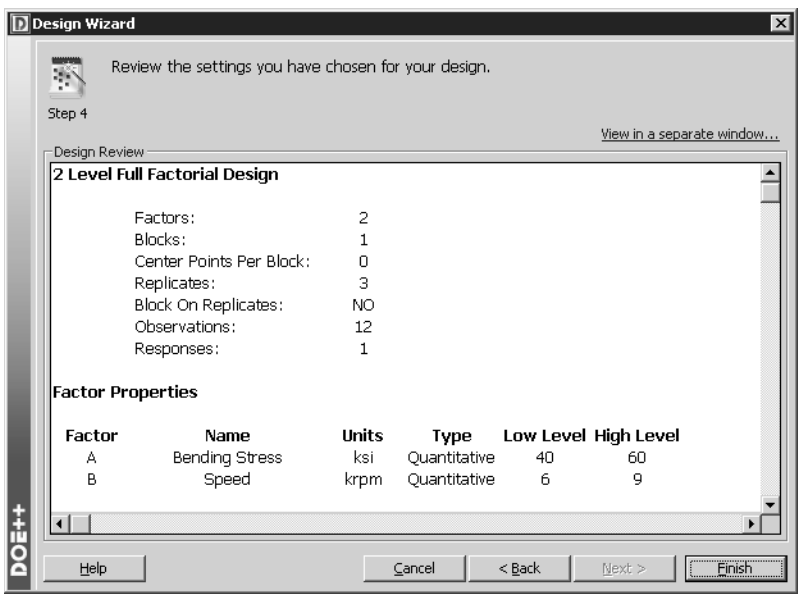

  Note that these factors are both quantitative (*i.e.* results can be generalized to values other than the ones that you provide here), not qualitative (*i.e.* results would be valid only for the values that you provide here), so leave the **Quantitative** entry in the **Type** column for both.

- Click **OK** to close the Factor Properties window and return to the Design Wizard.

- Click the **Response Properties** button. In the Response Properties window that appears, define the response name, as shown next.




- Click **OK** to close the Response Properties window and return to the Design Wizard.
- Click **Next** to proceed to the next step.
- The fourth step of the Design Wizard provides a review of the settings you have chosen for the design.

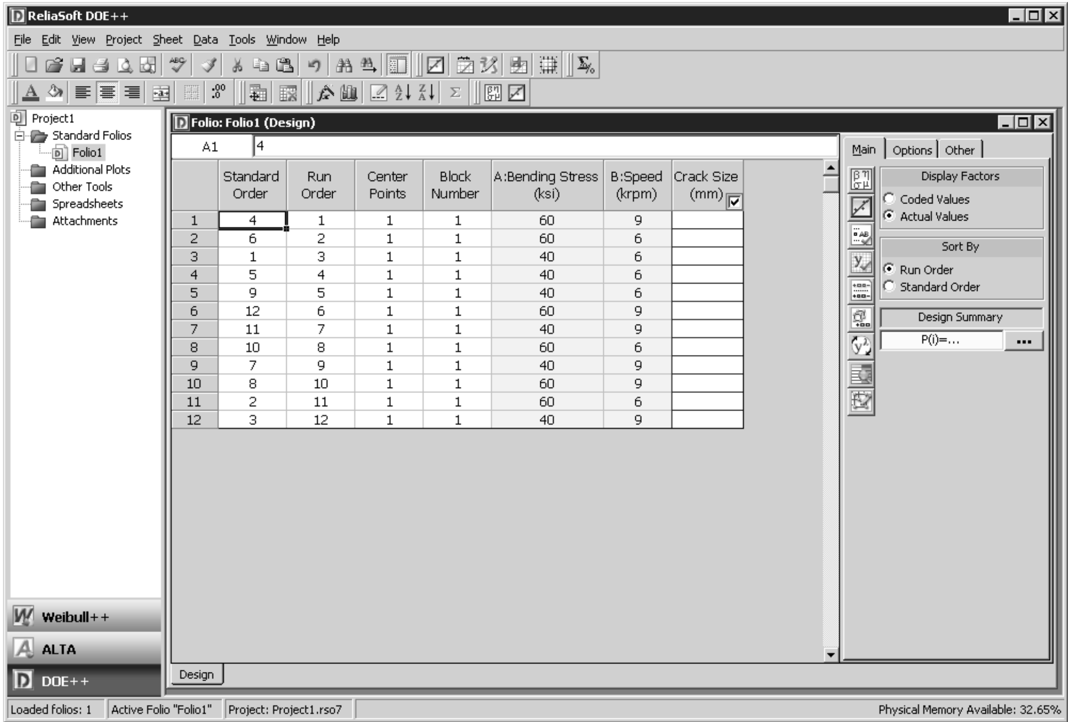



If any of the settings are not as intended, you can click the **Back** button to return to the appropriate step and make any necessary changes.[1]

- Click **Finish** to create the new project with the Standard Folio containing the two level full factorial design, as shown next. Note that by default, the Design tab Data Sheet is sorted by the Run Order column, which is the randomized order, generated by DOE++, in which it is recommended to perform

1. Note that you can skip the design review step by clicking **Finish** after completing the previous step, rather than clicking **Next**.

the runs to avoid biased results. Because the order is random, your Data Sheet may not be in the same order as the one shown next.

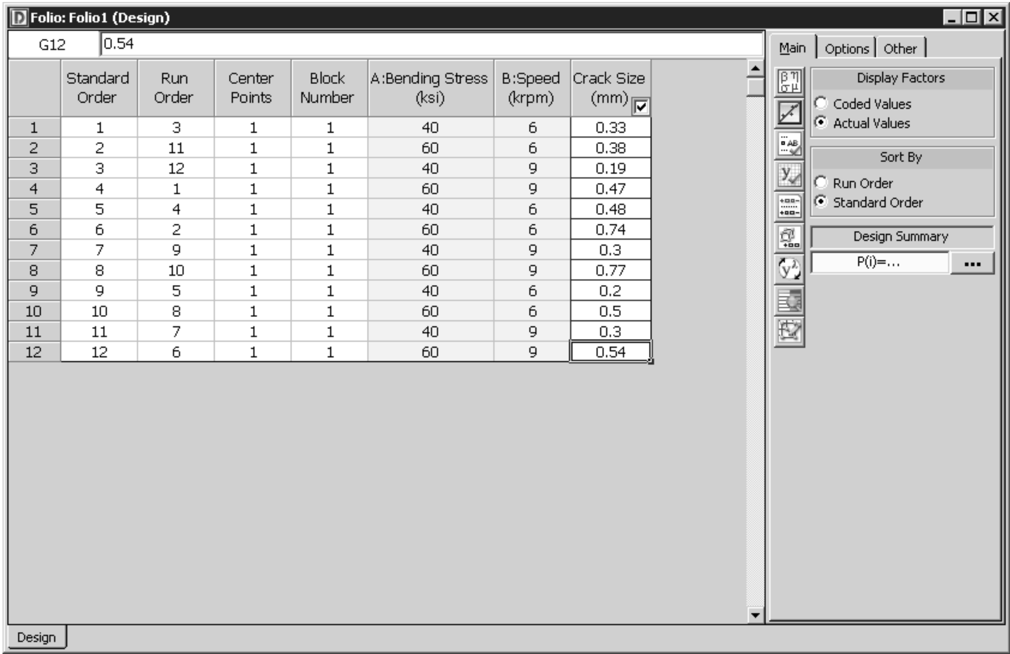



Upon creation, the Standard Folio contains only the Design tab. Notice that the factor columns are shown in green and are populated with factor settings.

### 3.4.2 Entering the Data

- For ease of entry, the data in this example have been presented in the standard order as specified in the design type. To sort the Data Sheet by the Standard Order column, select **Standard Order** in the **Sort By** area in the Control Panel.

- You can now enter the response data (*i.e.* Crack Size) in the order given in this example on page 9. The Folio will look like the one shown next.



Folio: Folio1 (Design)

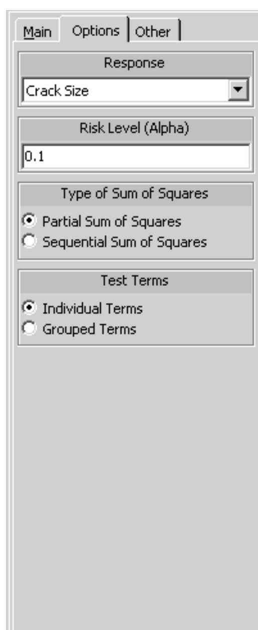
G12 | 0.54

| | Standard Order | Run Order | Center Points | Block Number | A:Bending Stress (ksi) | B:Speed (krpm) | Crack Size (mm) |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 3 | 1 | 1 | 40 | 6 | 0.33 |
| 2 | 2 | 11 | 1 | 1 | 60 | 6 | 0.38 |
| 3 | 3 | 12 | 1 | 1 | 40 | 9 | 0.19 |
| 4 | 4 | 1 | 1 | 1 | 60 | 9 | 0.47 |
| 5 | 5 | 4 | 1 | 1 | 40 | 6 | 0.48 |
| 6 | 6 | 2 | 1 | 1 | 60 | 6 | 0.74 |
| 7 | 7 | 9 | 1 | 1 | 40 | 9 | 0.3 |
| 8 | 8 | 10 | 1 | 1 | 60 | 9 | 0.77 |
| 9 | 9 | 5 | 1 | 1 | 40 | 6 | 0.2 |
| 10 | 10 | 8 | 1 | 1 | 60 | 6 | 0.5 |
| 11 | 11 | 7 | 1 | 1 | 40 | 9 | 0.3 |
| 12 | 12 | 6 | 1 | 1 | 60 | 9 | 0.54 |



## 3.4.3 Performing the Analysis

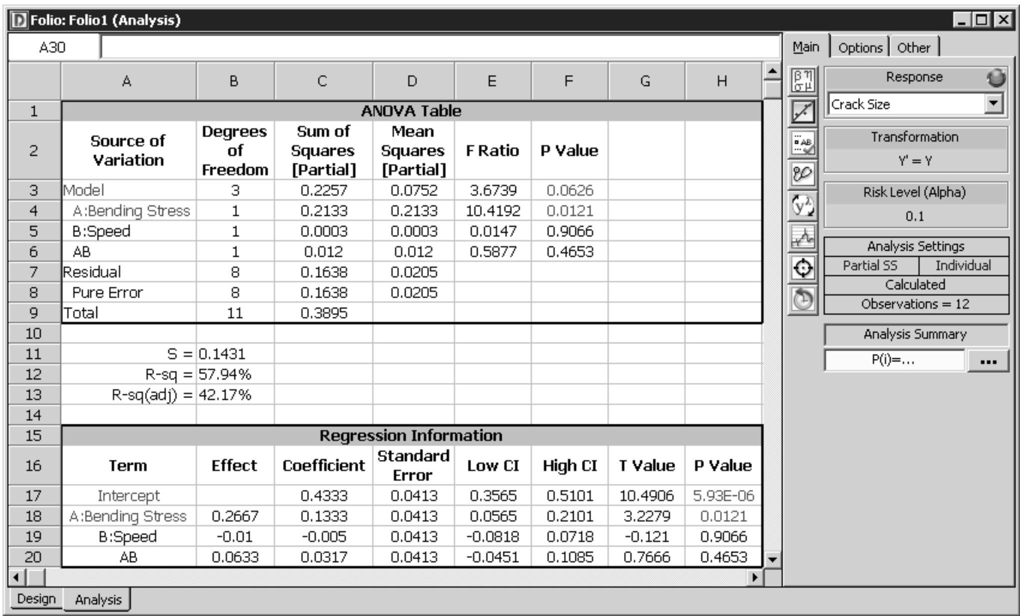
- Click the **Options** tab in the Control Panel.
- Specify a **Risk Level (Alpha)** value of **0.1** for the Crack Size response and select to use **Individual Terms** in the **Test Terms** area. Leave **Partial Sum of Squares** selected in the **Type of Sum of Squares** area, as shown next.



The risk level is a measure of the risk that the analytical conclusions for the response are incorrect (*i.e.* alpha = 1 - confidence level).[2] The test terms specify what the analysis will examine; using individual terms causes DOE++ to examine the effects of each individual factor and/or factorial interaction separately.[3]

- Click the **Main** tab in the Control Panel and then click the **Calculate** icon to perform the analysis.

The Analysis tab will be added to the Standard Folio, as shown next.




| ANOVA Table | | | | | |
|---|---|---|---|---|---|
| Source of Variation | Degrees of Freedom | Sum of Squares [Partial] | Mean Squares [Partial] | F Ratio | P Value |
| Model | 3 | 0.2257 | 0.0752 | 3.6739 | 0.0626 |
| A:Bending Stress | 1 | 0.2133 | 0.2133 | 10.4192 | 0.0121 |
| B:Speed | 1 | 0.0003 | 0.0003 | 0.0147 | 0.9066 |
| AB | 1 | 0.012 | 0.012 | 0.5877 | 0.4653 |
| Residual | 8 | 0.1638 | 0.0205 | | |
| Pure Error | 8 | 0.1638 | 0.0205 | | |
| Total | 11 | 0.3895 | | | |

S = 0.1431
R-sq = 57.94%
R-sq(adj) = 42.17%

| Regression Information | | | | | | | |
|---|---|---|---|---|---|---|---|
| Term | Effect | Coefficient | Standard Error | Low CI | High CI | T Value | P Value |
| Intercept | | 0.4333 | 0.0413 | 0.3565 | 0.5101 | 10.4906 | 5.93E-06 |
| A:Bending Stress | 0.2667 | 0.1333 | 0.0413 | 0.0565 | 0.2101 | 3.2279 | 0.0121 |
| B:Speed | -0.01 | -0.005 | 0.0413 | -0.0818 | 0.0718 | -0.121 | 0.9066 |
| AB | 0.0633 | 0.0317 | 0.0413 | -0.0451 | 0.1085 | 0.7666 | 0.4653 |

The Analysis tab Data Sheet contains information that describes how each factor and factorial interaction, or "source," affects the variation of the response that is currently selected in the Control Panel. In this example, we are interested in which factor(s) significantly affect the response and whether or not the interaction between the two factors is significant. This can be determined from the P Value column in the ANOVA (Analysis of Variance) table, which displays for each source the probability that an equal amount of variation in the output would be observed in the case that the source does not affect the output. This value is compared to the risk level (alpha) that you specified on the Options page of the Control Panel. If the *p* value is less than alpha, this source of variation is considered to have a significant effect on the output and the *p* value will be displayed in red.

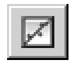
Note that the *p* value for factor A, bending stress, is red but the *p* values for factor B, speed, and the interaction between bending stress and speed (AB) are black. This indicates that at a significance level of 10%, bending stress is the only factor in the test that has a significant effect on the response and that speed does not interact significantly with bending stress.

2. Note that the risk level can be specified separately for each response if more than one response is used in the design.

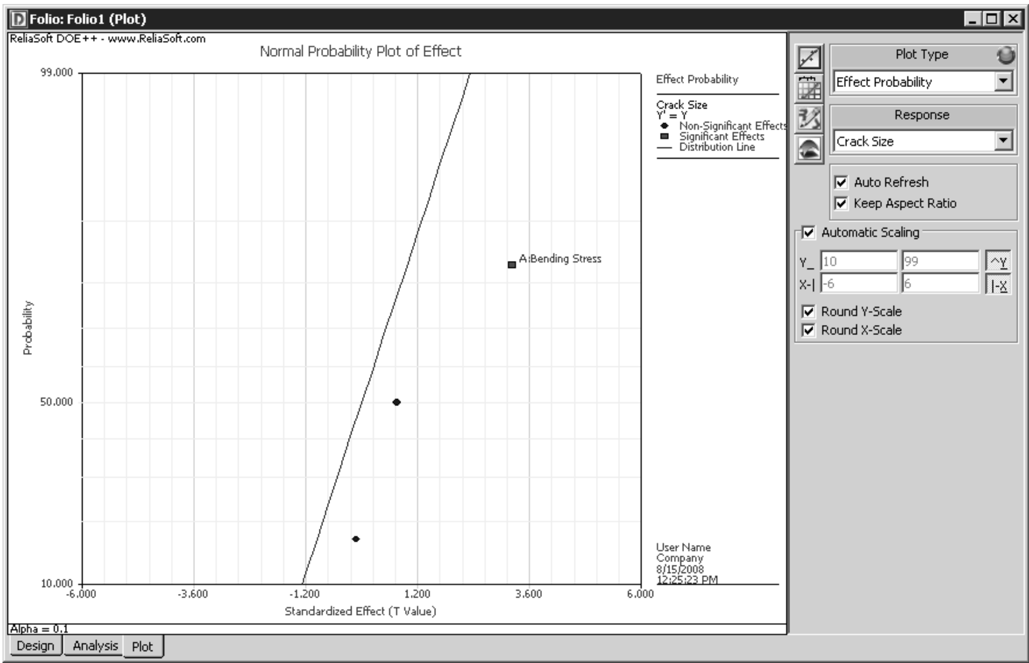
3. Using grouped terms causes DOE++ to examine groups of effect types, such as main effects (*e.g.* A, B, C...), order 2 effects (*e.g.* AB, AC, BC...), etc.

### 3.4.4 Plotting the Data

- Click the **Plot** icon to create a plot based on the analysis.



The Plot tab will be added to the Standard Folio. The Effect Probability plot will be displayed by default, as shown next.[4]

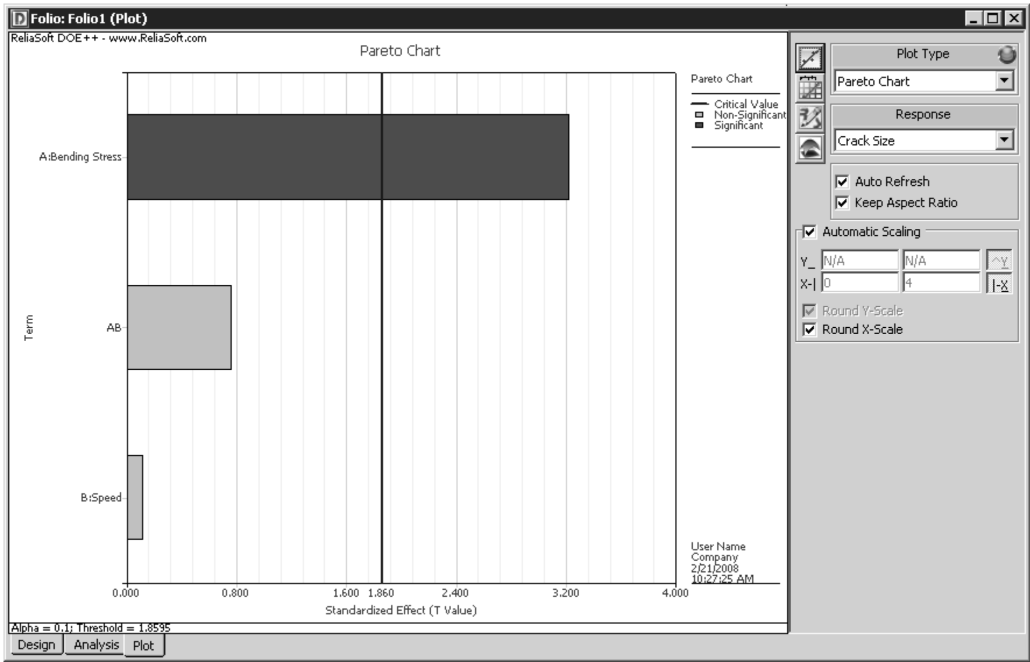



The Effect Probability plot is a linear representation of probability versus the standardized effect for the currently selected response (*i.e.* the probability that any term's standardized effect will be lower than the given value). The points on this plot represent the values for each term as shown in the T Value column of the Regression Information table on the Analysis tab.

- Select **Pareto Chart** from the **Plot Type** menu in the Control Panel. If the **Auto Refresh** command is enabled, the display will be updated automatically when you make changes in the Control Panel. If not,

4. If necessary, you may wish to re-size the plot by re-sizing the Folio.

select **Redraw Plot** from the **Plot** menu or click the **Redraw Plot** icon to implement your selections. The Pareto chart will be displayed, as shown next.




The Pareto chart shows the standardized effect of each term (*i.e.* factor or combination of factors) for the currently selected response. The thick blue line is the threshold value. If a bar is beyond the blue line, it will be red, indicating that this effect is significant. You can see in this case that factor A, bending stress, is significant and that factor B, speed, and the interaction between factors A and B are not significant.

Notice that the analysis settings are displayed in the legend in the top right corner of the plot and the alpha (risk level) and calculated threshold values are displayed in the bottom left corner. You can customize these and other display settings from the Plot Setup window. To access the Plot Setup, select **Plot Setup** from the **Plot** menu or click the **Plot Setup** icon.

- When you are finished experimenting with the Plot Setup window, save the project by selecting **Save** from the **File** menu or by clicking the **Save** icon.

- When prompted to specify the name and location for the file, browse to the directory of your choice and type "1stSteps" for the File name.[5] Accept the default type (*.rso7) and click **Save** to close the window and save the file.
- Close the project by selecting **Close** from the **File** menu. You will now be looking at the MDI with no project open.

5. By default, files will be saved in the "My Documents" directory on your computer. You can select a different directory, if desired, and DOE++ will "remember" the directory for the next time that you save a file.

# Step-by-Step Examples 4

This chapter presents the following examples:



## 4.1 Example 1: One Factor Design

An engineer is interested in studying the effect of different corrosion inhibitors on the rate of corrosion of steel. Three NaCl aqueous solutions containing different inhibitors (aluminum, calcium and magnesium cations) are used, as well as one NaCl aqueous solution with no inhibitors. The samples are tested over the time interval of interest.

The corrosion rate ($\mu gm^{-2}s^{-1}$) is measured in 20 samples using randomization. The measurements are shown next.

| (NaCl) | (NaCl + Mg2+) | (NaCl + Ca2+) | (NaCl + Al3+) |
|---|---|---|---|
| 8.1 | 6.9 | 4.6 | 4.8 |
| 7.9 | 7.2 | 4.2 | 3.8 |
| 8.6 | 6.6 | 4.3 | 3.9 |
| 8.9 | 6.5 | 4.7 | 4.5 |
| 8.3 | 7.1 | 5.1 | 4.1 |

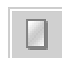
Do the following:

- If the model accurately accounts for the effect of the factor, the residuals (*i.e.* the differences between the observed response values at each factor level and the response values predicted by the model) should be approximately normally and independently distributed. Graphically validate the normality assumption.
- Determine whether there is a significant difference between treatments at a 10% significance level.

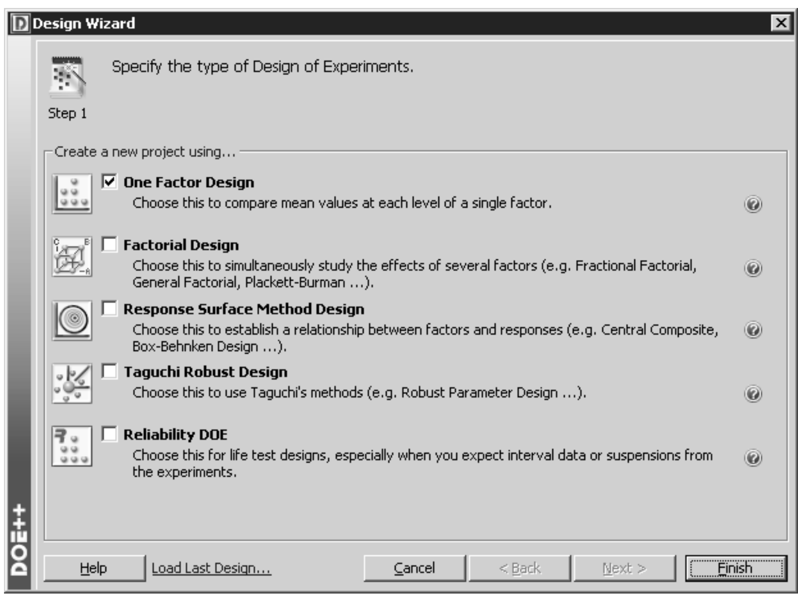
*The sample file for this example is located in the "Training Guide" folder in your application directory (e.g. C:\Program Files\ReliaSoft\DOE\Training Guide) and is named "OneFactor.rdoe." The data set for this example is available in the Spreadsheet in the "OneFactor-Data.rso7" file located in the "Raw Data" folder within the "Training Guide" folder.*

**Solution**

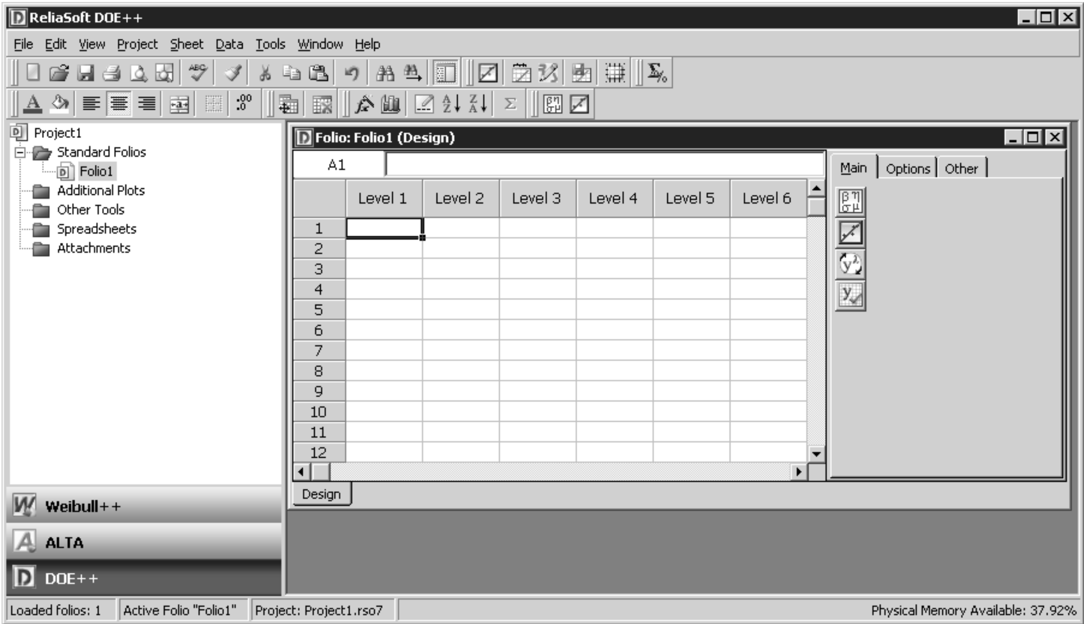
- Create a new project by clicking **Create a New Project** in the initial window that may appear at startup, by selecting **New** from the **File** menu or by clicking the **New** icon.



- Select **One Factor Design** on the first page of the Design Wizard, as shown next.

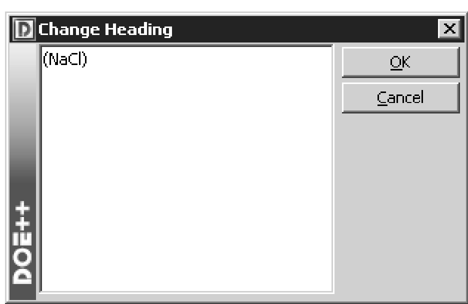


- Click **Finish** to create the new project with the Standard Folio containing the one factor design, as shown next.



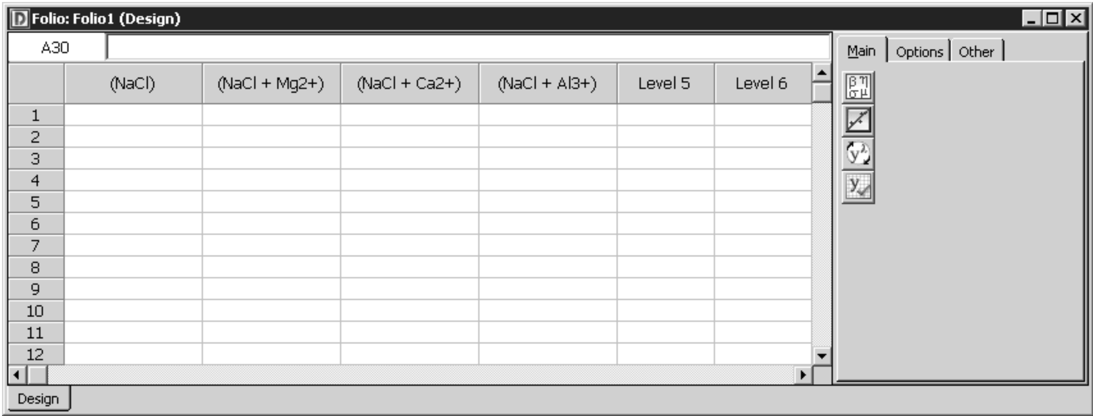

- Save the project by selecting **Save** from the **File** menu or by clicking the **Save** icon.

- When prompted to specify the name and location for the file, browse to the directory of your choice and type "OneFactor" for the File name.[1] Accept the default type (*.rso7) and click **Save** to close the window and save the file.
- To change the title of the first column, double-click the column header and, in the window that appears, type **(NaCl)**, as shown next, then click **OK**.

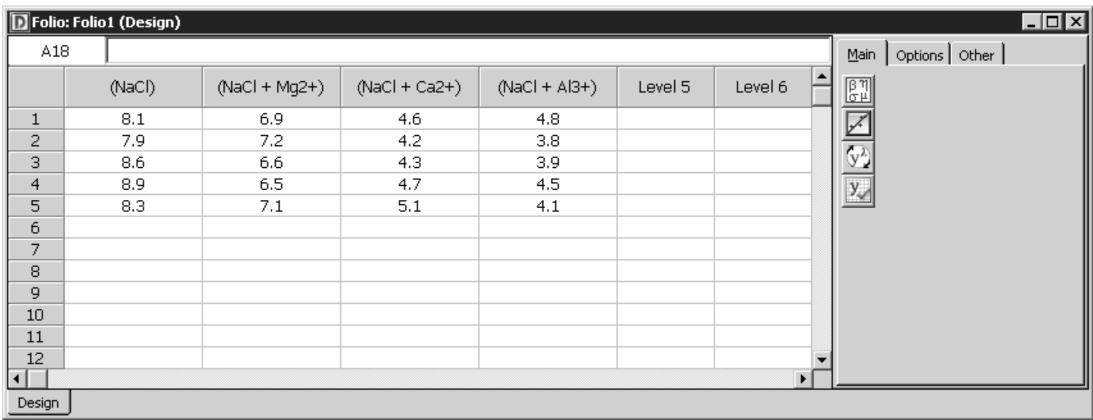



[1] By default, files will be saved in the "My Documents" directory on your computer. You can select a different directory, if desired, and DOE++ will "remember" the directory for the next time that you save a file.

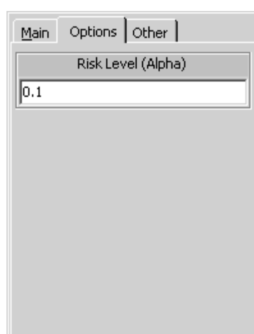
- Do the same for the second, third and fourth columns, titling them **(NaCl + Mg2+)**, **(NaCl + Ca2+)** and **(NaCl + Al3+)**, respectively. When you have finished renaming the columns, the Folio will look like the one shown next.[2]




- Enter the data, as shown next.



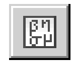

- Click the **Options** tab in the Control Panel and specify a **Risk Level (Alpha)** value of **0.1**, as shown next.



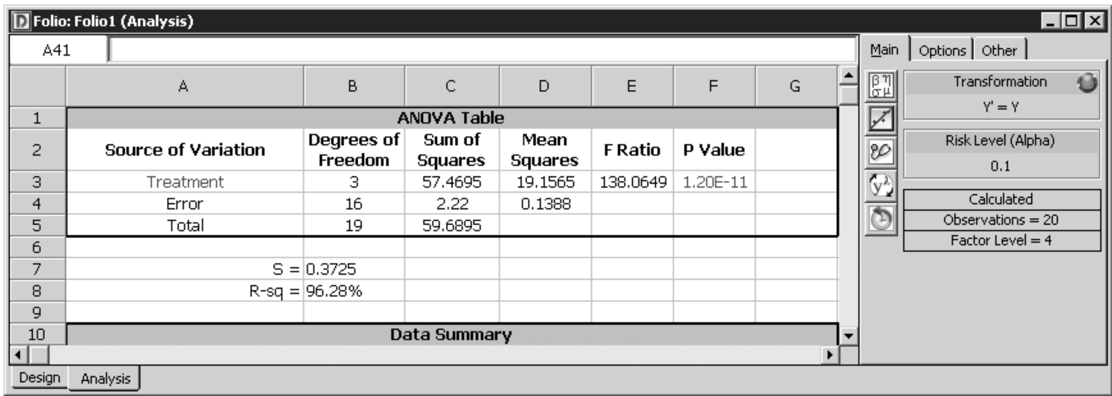

The risk level is a measure of the risk that the analytical conclusions for the response are incorrect (*i.e.* alpha = 1 - confidence level).

2. Note that the columns in the Folio shown here have been widened to display the full column name. To do this, you can click and drag the right-hand border of each column header.

- Click the **Main** tab in the Control Panel and then click the **Calculate** icon to perform the analysis.

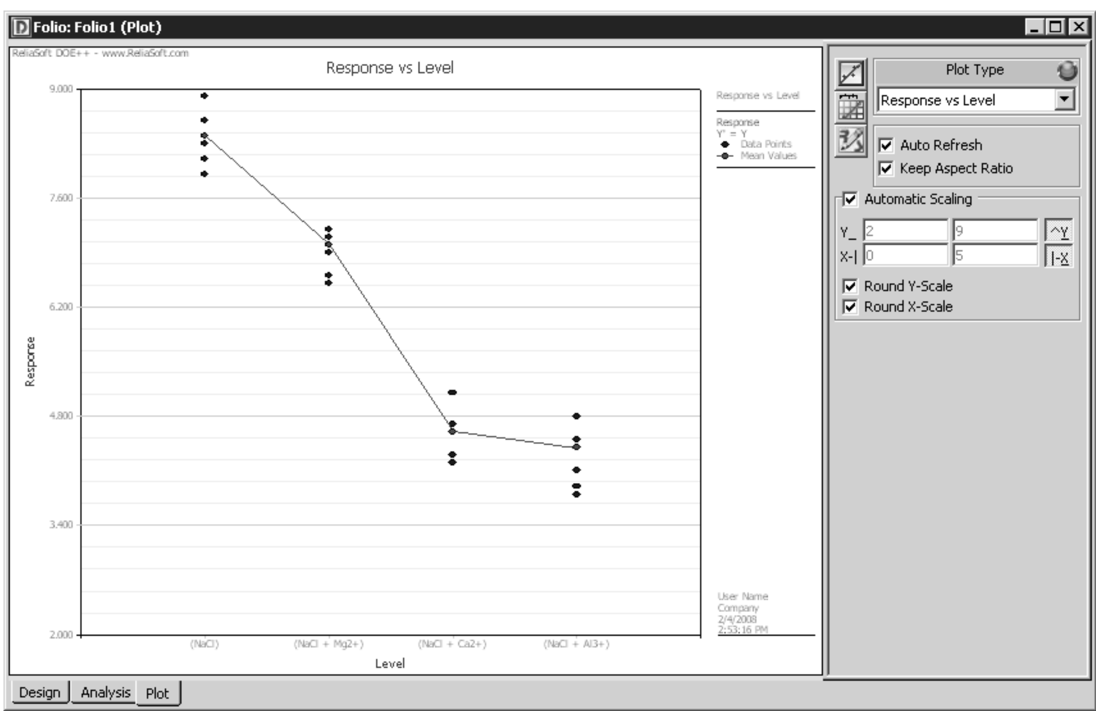


Note that if desired, you can click the **Calculate** icon in the Data Analysis Tools toolbar or select **Calculate** from the Data menu instead. The Analysis tab will be added to the Standard Folio, as shown next.

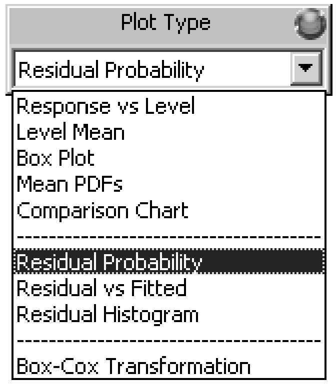



- To validate the normality assumption graphically, we will use the Residual Probability plot. Plot the data by selecting **Plot** from the **Data** menu or by clicking the **Plot** icon.

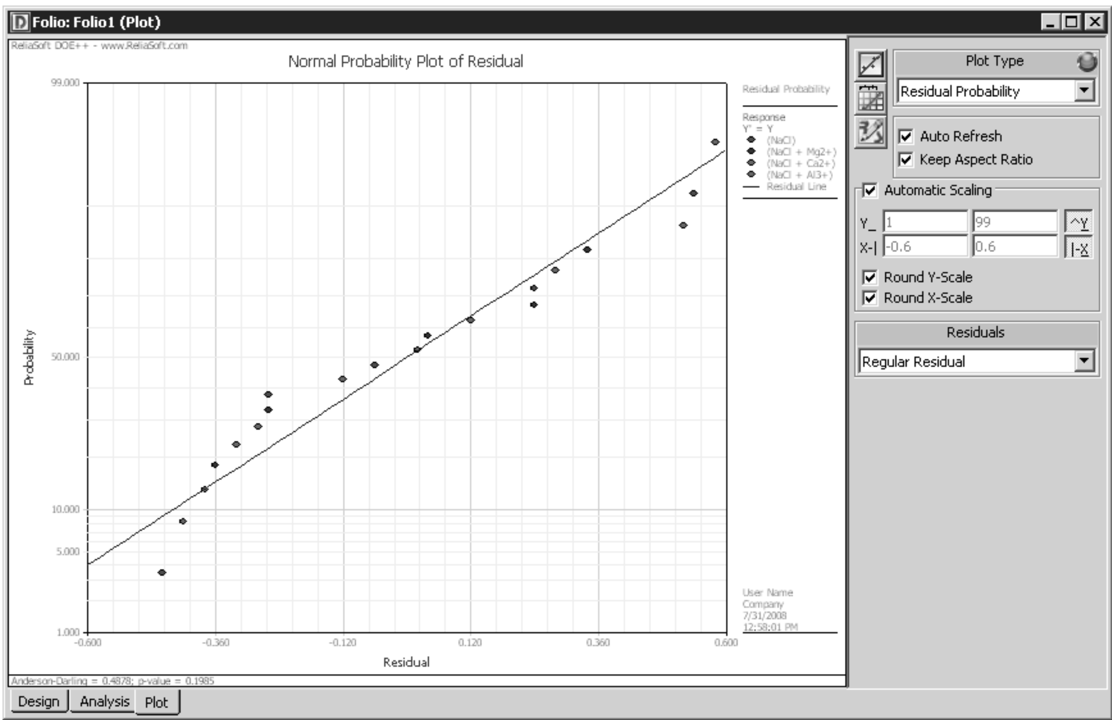
The Plot tab will be added to the Standard Folio. The Response vs. Level plot will be displayed by default, as shown next.



- Select **Residual Probability** from the **Plot Type** menu in the Control Panel.



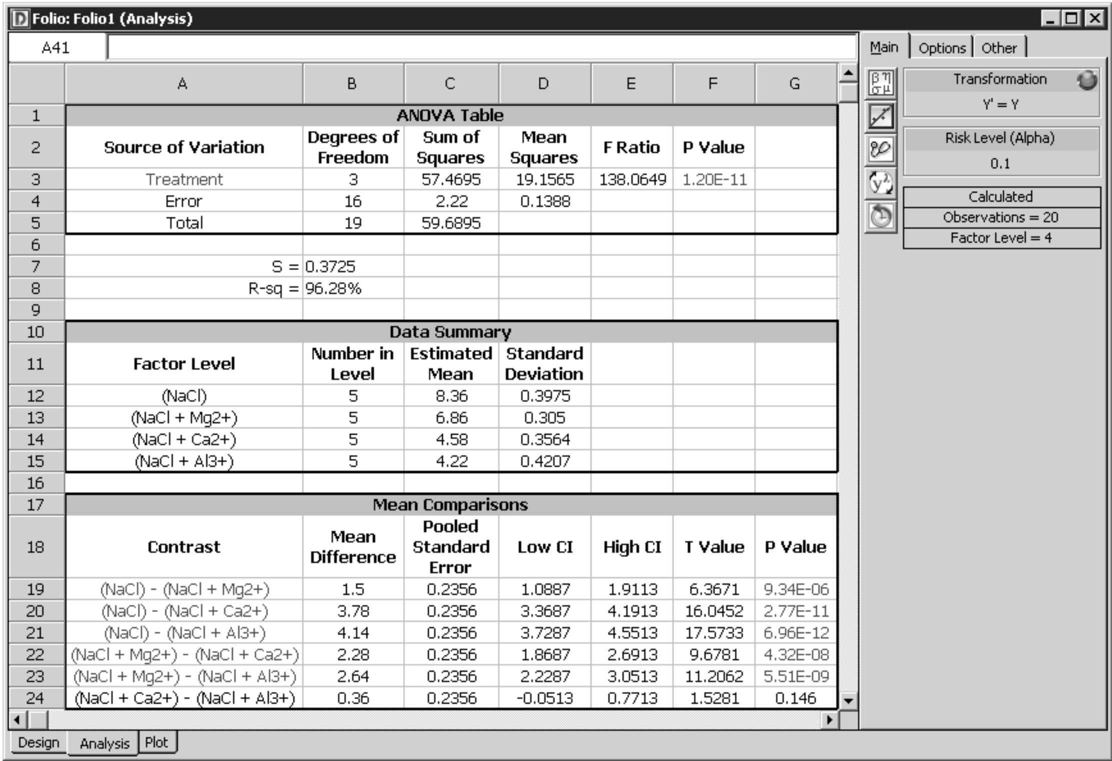

The Residual Probability plot will be displayed, as shown next.[3]




The data points follow the line reasonably closely, indicating that the normality assumption is valid.

3. If the **Auto Refresh** command is enabled, the display will be updated automatically when you make changes in the Control Panel. If not, select **Redraw Plot** from the **Plot** menu or click the **Redraw Plot** icon to implement your selections.

- Return to the Analysis tab by clicking its page index tab. The complete Analysis tab is shown next.


Folio: Folio1 (Analysis)

A41

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | ANOVA Table | | | | | | |
| 2 | Source of Variation | Degrees of Freedom | Sum of Squares | Mean Squares | F Ratio | P Value | |
| 3 | Treatment | 3 | 57.4695 | 19.1565 | 138.0649 | 1.20E-11 | |
| 4 | Error | 16 | 2.22 | 0.1388 | | | |
| 5 | Total | 19 | 59.6895 | | | | |
| 6 | | | | | | | |
| 7 | S = | 0.3725 | | | | | |
| 8 | R-sq = | 96.28% | | | | | |
| 9 | | | | | | | |
| 10 | Data Summary | | | | | | |
| 11 | Factor Level | Number in Level | Estimated Mean | Standard Deviation | | | |
| 12 | (NaCl) | 5 | 8.36 | 0.3975 | | | |
| 13 | (NaCl + Mg2+) | 5 | 6.86 | 0.305 | | | |
| 14 | (NaCl + Ca2+) | 5 | 4.58 | 0.3564 | | | |
| 15 | (NaCl + Al3+) | 5 | 4.22 | 0.4207 | | | |
| 16 | | | | | | | |
| 17 | Mean Comparisons | | | | | | |
| 18 | Contrast | Mean Difference | Pooled Standard Error | Low CI | High CI | T Value | P Value |
| 19 | (NaCl) - (NaCl + Mg2+) | 1.5 | 0.2356 | 1.0887 | 1.9113 | 6.3671 | 9.34E-06 |
| 20 | (NaCl) - (NaCl + Ca2+) | 3.78 | 0.2356 | 3.3687 | 4.1913 | 16.0452 | 2.77E-11 |
| 21 | (NaCl) - (NaCl + Al3+) | 4.14 | 0.2356 | 3.7287 | 4.5513 | 17.5733 | 6.96E-12 |
| 22 | (NaCl + Mg2+) - (NaCl + Ca2+) | 2.28 | 0.2356 | 1.8687 | 2.6913 | 9.6781 | 4.32E-08 |
| 23 | (NaCl + Mg2+) - (NaCl + Al3+) | 2.64 | 0.2356 | 2.2287 | 3.0513 | 11.2062 | 5.51E-09 |
| 24 | (NaCl + Ca2+) - (NaCl + Al3+) | 0.36 | 0.2356 | -0.0513 | 0.7713 | 1.5281 | 0.146 |

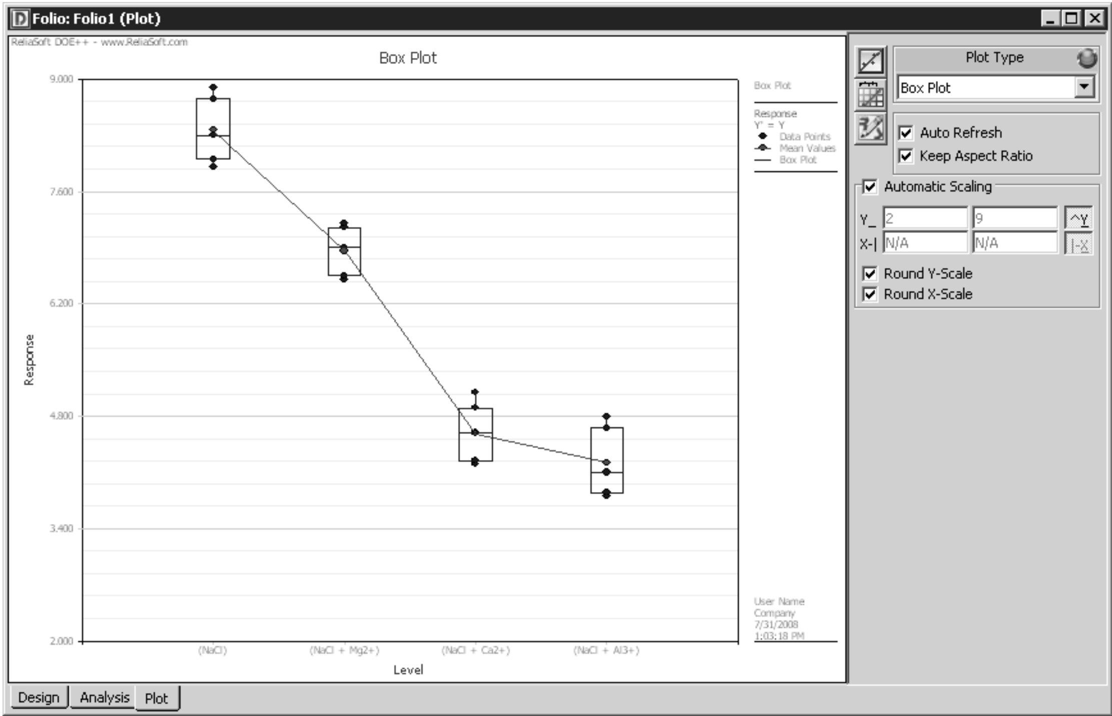


In the ANOVA table, the treatment and its *p* value are displayed in red, indicating that there is a significant difference in the corrosion rate when using the different solutions. In addition, the mean pair comparisons displayed in the Mean Comparisons table show that individual pairs are also significantly different, except for the solutions using calcium and aluminum cations.

A similar conclusion can be reached by examining the Box plot.

- Return to the Plot tab by clicking its page index tab.

- Select **Box Plot** from the **Plot Type** menu in the Control Panel. The Box plot is shown next.




Graphically, the overlap between the solutions containing calcium and aluminum indicate that there is no significant difference between them, while the solution containing magnesium and the control solution with no inhibitor show significant difference.

- Save the project by selecting **Save** from the **File** menu or by clicking the **Save** icon.
- Close the project by selecting **Close** from the **File** menu and proceed to the next example.

## 4.2 Example 2: Two Level Full Factorial Design

Consider a three-factor experiment to investigate the effects of honing pressure, number of strokes and cycle time on the surface finish of automobile brake drums. Each of these factors is investigated at two levels. The honing pressure is investigated at levels of 200 psi and 400 psi; the two levels of the number of strokes used are 3 and 5 and the two levels of the cycle time are 3 and 5 seconds. It is decided to run two replicates for this experiment. (The number of replicates is the number of times you want to perform the complete set of runs required by the design.) The surface finish values obtained are converted to a dimensionless index with values ranging from 0 to 150. The data set is shown next.

| Honing Pressure (psi) | No. of Strokes | Cycle Time (sec) | Surface Finish |
|---|---|---|---|
| 200 | 3 | 3 | 90 |
| 400 | 3 | 3 | 90 |
| 200 | 5 | 3 | 85 |
| 400 | 5 | 3 | 80 |
| 200 | 3 | 5 | 90 |

| Honing Pressure (psi) | No. of Strokes | Cycle Time (sec) | Surface Finish |
|---|---|---|---|
| 400 | 3 | 5 | 105 |
| 200 | 5 | 5 | 75 |
| 400 | 5 | 5 | 90 |
| 200 | 3 | 3 | 86 |
| 400 | 3 | 3 | 90 |
| 200 | 5 | 3 | 80 |
| 400 | 5 | 3 | 82 |
| 200 | 3 | 5 | 85 |
| 400 | 3 | 5 | 95 |
| 200 | 5 | 5 | 80 |
| 400 | 5 | 5 | 80 |

Do the following:

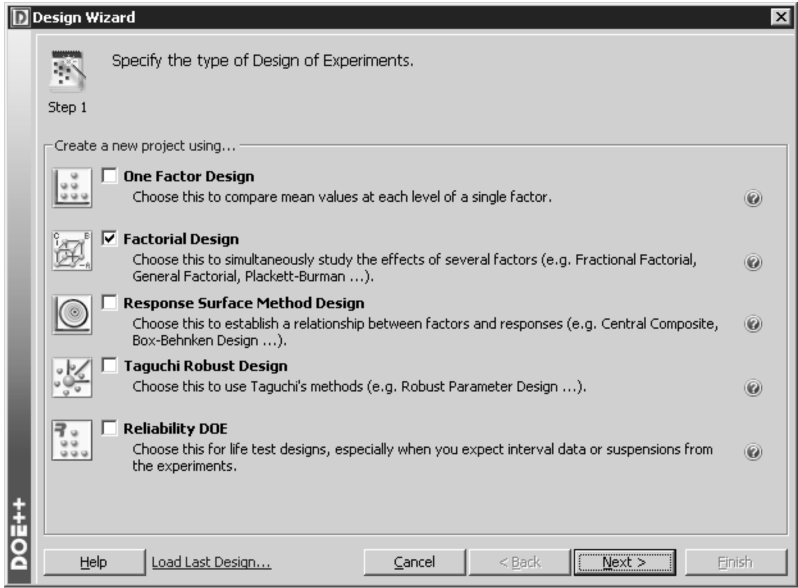
- Determine which factors and/or factorial interactions have a significant effect on the response.
- Graphically validate the normality assumption.
- Determine the parameters of the model.

*The sample file for this example is located in the "Training Guide" folder in your application directory (e.g. C:\Program Files\ReliaSoft\DOE\Training Guide) and is named "TwoLevelFullFactorial.rdoe." The data set for this example is available in the Spreadsheet in the "TwoLevelFullFactorial-Data.rso7" file located in the "Raw Data" folder within the "Training Guide" folder.*

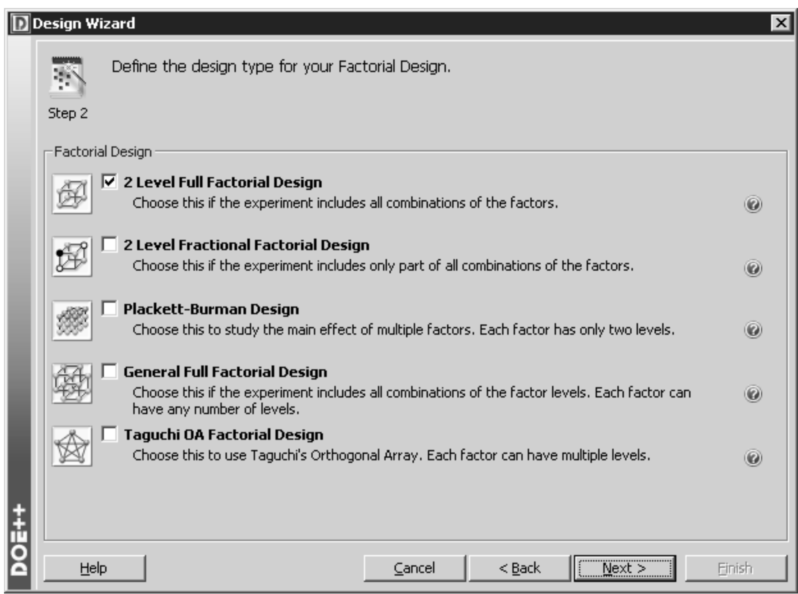
**Solution**

- Create a new project by clicking **Create a New Project** in the initial window that may appear at startup, by selecting **New** from the **File** menu or by clicking the **New** icon.

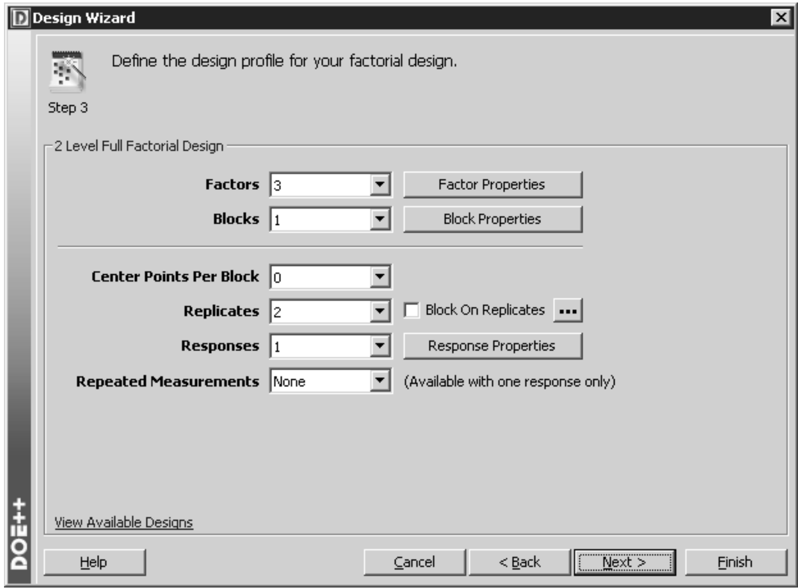
- On the first page of the Design Wizard, select to create a **Factorial Design** and click **Next** to proceed to the next step.



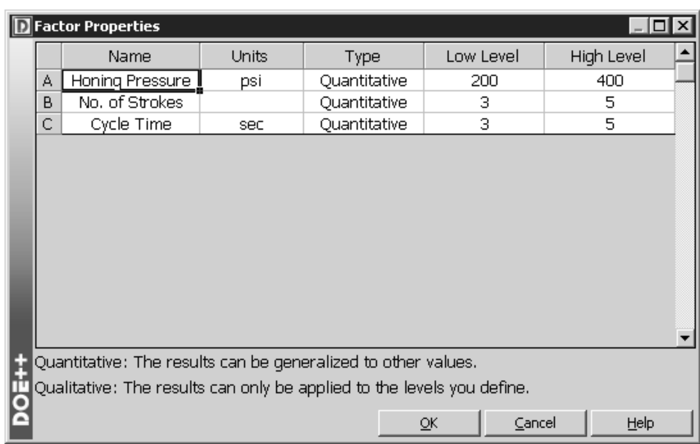

- On the second page of the Design Wizard, select **2 Level Full Factorial Design** and click **Next** to proceed to the next step.



- The third step of the Design Wizard allows you to define the exact configuration of the design. Select **3** from the **Factors** drop-down and **2** from the **Replicates** drop-down and leave all other fields at their default settings, as shown next.




- Click the **Factor Properties** button. In the Factor Properties window that appears, enter the names, units and high and low level values for the factors, as shown next.

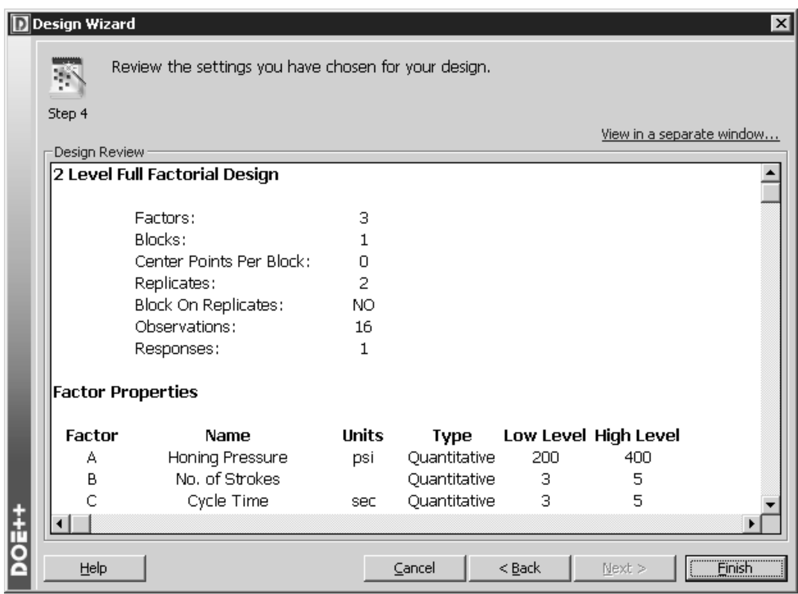



Note that these factors are all quantitative (*i.e.* results can be generalized to values other than the ones that you provide here), not qualitative (results would be valid only for the values that you provide here), so leave the **Quantitative** entry in the **Type** column for all three factors.

- Click **OK** to close the Factor Properties window and return to the Design Wizard.

- Click the **Response Properties** button. In the Response Properties window that appears, define the response name, as shown next.


- Click **OK** to close the Response Properties window and return to the Design Wizard.
- Click **Next** to proceed to the next step.
- The fourth step of the Design Wizard provides a review of the settings you have chosen for the design.

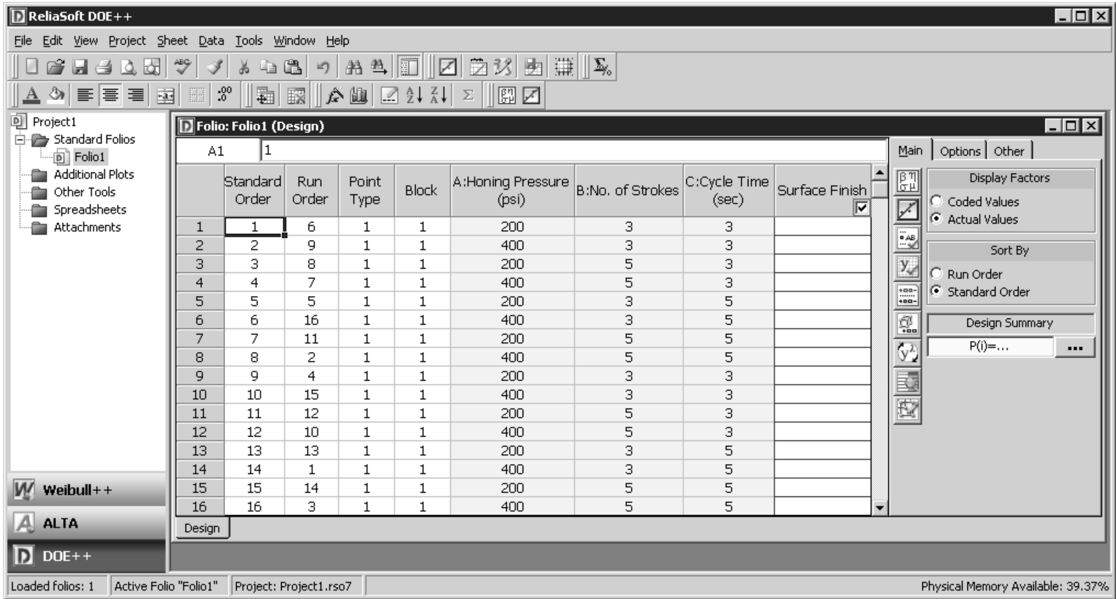



If any of the settings are not as intended, you can click the **Back** button to return to the appropriate step and make any necessary changes.

- Click **Finish** to create the new project with the Standard Folio containing the two level full factorial design.

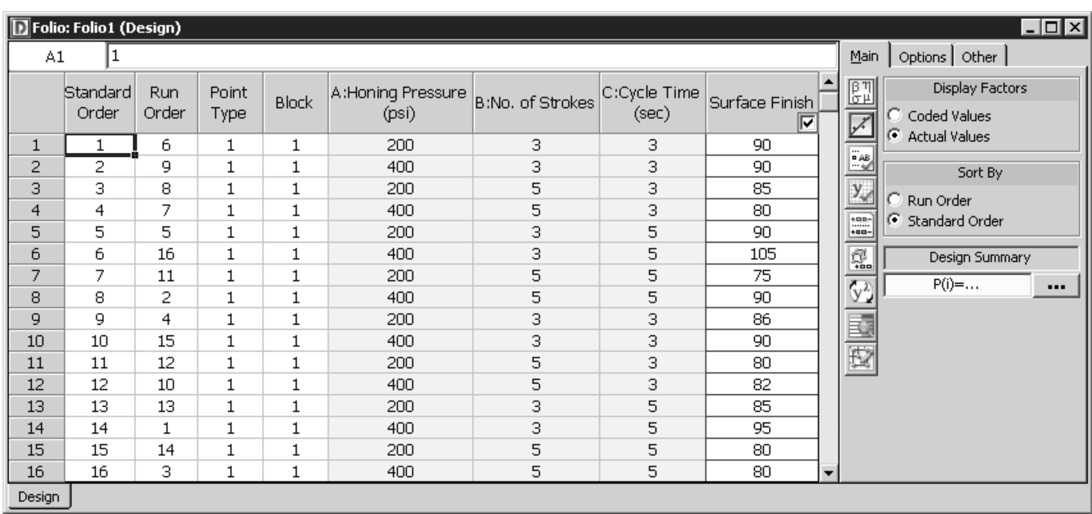
By default, the Design tab Data Sheet is sorted by the Run Order column, which is the randomized order, generated by DOE++, in which it is recommended to perform the runs to avoid biased results. For ease of entry, the data in this example have been presented in the standard order as specified in the design type.

- To sort the Data Sheet by the Standard Order column, select **Standard Order** in the **Sort By** area in the Control Panel. The Standard Folio will look like the one shown next.



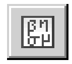

- Save the project by selecting **Save** from the **File** menu or by clicking the **Save** icon.

- When prompted to specify the name and location for the file, browse to the directory of your choice and type "TwoLevelFullFactorial" for the File name. Accept the default type (*.rso7) and click **Save** to close the window and save the file.
- You can now enter the response data in the order given in this example on page 26. The Folio will look like the one shown next.

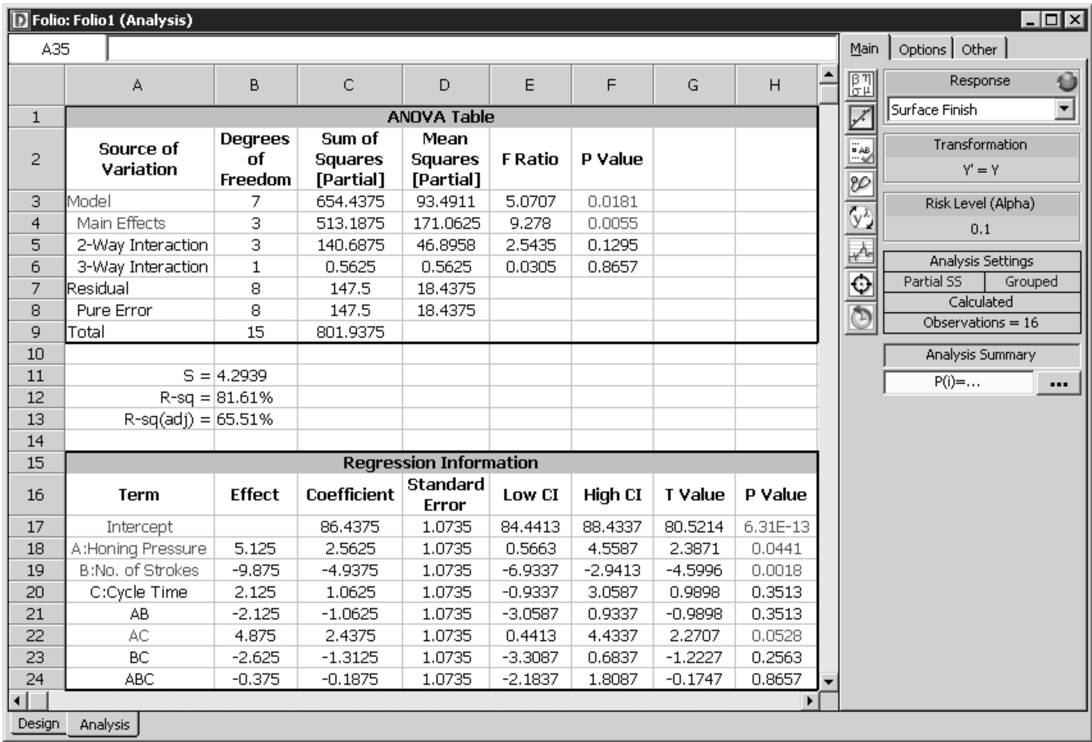



- Click the **Options** tab in the Control Panel to view the Options page of the Control Panel.

- Specify a **Risk Level (Alpha)** value of **0.1** for the Surface Finish response and select to use **Grouped Terms** in the **Test Terms** area.[4]
- Return to the Main page of the Control Panel and click the **Calculate** icon to perform the analysis.



The Analysis tab will be added to the Standard Folio, as shown next.



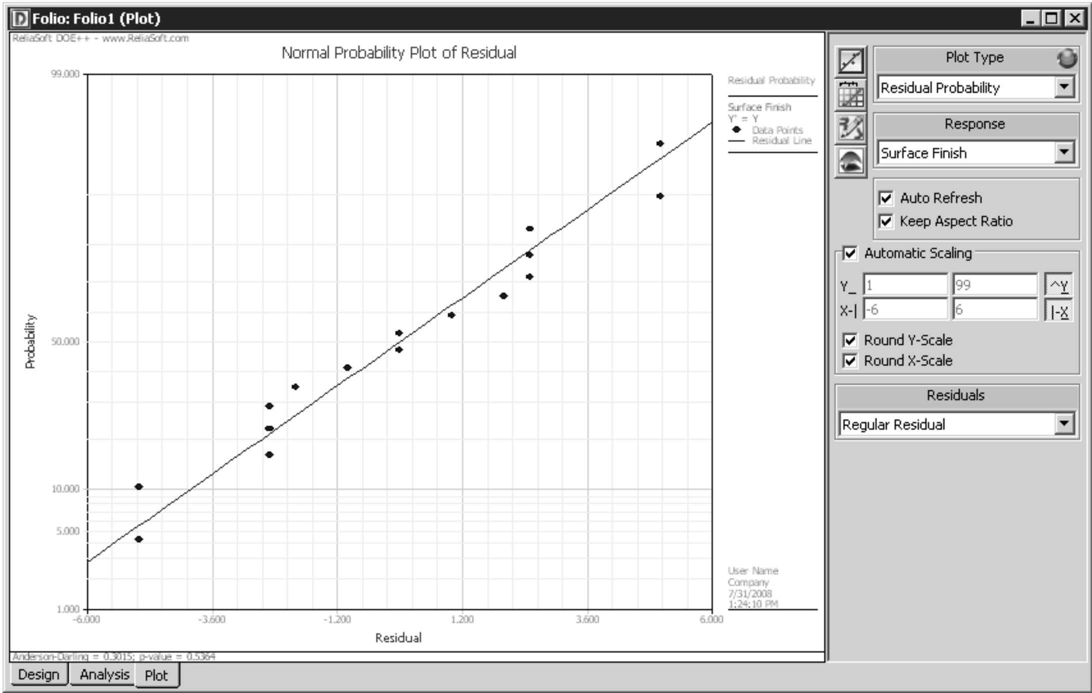

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | ANOVA Table | | | | | | | |
| 2 | Source of Variation | Degrees of Freedom | Sum of Squares [Partial] | Mean Squares [Partial] | F Ratio | P Value | | |
| 3 | Model | 7 | 654.4375 | 93.4911 | 5.0707 | 0.0181 | | |
| 4 | Main Effects | 3 | 513.1875 | 171.0625 | 9.278 | 0.0055 | | |
| 5 | 2-Way Interaction | 3 | 140.6875 | 46.8958 | 2.5435 | 0.1295 | | |
| 6 | 3-Way Interaction | 1 | 0.5625 | 0.5625 | 0.0305 | 0.8657 | | |
| 7 | Residual | 8 | 147.5 | 18.4375 | | | | |
| 8 | Pure Error | 8 | 147.5 | 18.4375 | | | | |
| 9 | Total | 15 | 801.9375 | | | | | |
| 10 | | | | | | | | |
| 11 | S = | 4.2939 | | | | | | |
| 12 | R-sq = | 81.61% | | | | | | |
| 13 | R-sq(adj) = | 65.51% | | | | | | |
| 14 | | | | | | | | |
| 15 | Regression Information | | | | | | | |
| 16 | Term | Effect | Coefficient | Standard Error | Low CI | High CI | T Value | P Value |
| 17 | Intercept | | 86.4375 | 1.0735 | 84.4413 | 88.4337 | 80.5214 | 6.31E-13 |
| 18 | A:Honing Pressure | 5.125 | 2.5625 | 1.0735 | 0.5663 | 4.5587 | 2.3871 | 0.0441 |
| 19 | B:No. of Strokes | -9.875 | -4.9375 | 1.0735 | -6.9337 | -2.9413 | -4.5996 | 0.0018 |
| 20 | C:Cycle Time | 2.125 | 1.0625 | 1.0735 | -0.9337 | 3.0587 | 0.9898 | 0.3513 |
| 21 | AB | -2.125 | -1.0625 | 1.0735 | -3.0587 | 0.9337 | -0.9898 | 0.3513 |
| 22 | AC | 4.875 | 2.4375 | 1.0735 | 0.4413 | 4.4337 | 2.2707 | 0.0528 |
| 23 | BC | -2.625 | -1.3125 | 1.0735 | -3.3087 | 0.6837 | -1.2227 | 0.2563 |
| 24 | ABC | -0.375 | -0.1875 | 1.0735 | -2.1837 | 1.8087 | -0.1747 | 0.8657 |

In the Regression Information table, note that factor A (honing pressure) and factor B (number of strokes) and the interaction between factors A and C (cycle time), along with their *p* values, are red, indicating that at a significance level of 10%, these effects are significant.

- To validate the normality assumption graphically, we will use the Residual Probability plot. Plot the data by selecting **Plot** from the **Data** menu or by clicking the **Plot** icon.

4. Note that these options can be specified separately for each response if more than one response is used in the design.

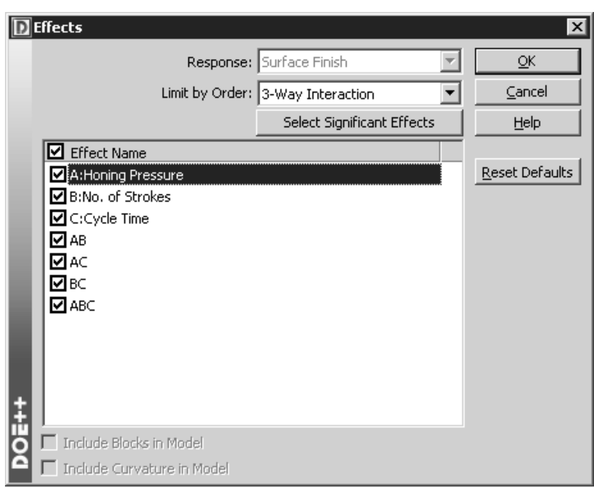
- Select **Residual Probability** from the **Plot Type** menu in the Control Panel. The Residual Probability plot will be displayed, as shown next.



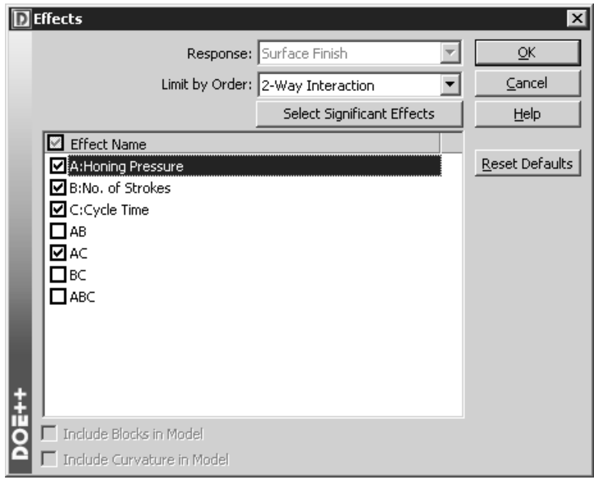

The data points follow the line reasonably closely, indicating that the normality assumption is valid.

- To determine the parameters of the model, we will need to analyze the data set including only the significant effects. To do this, return to the Analysis tab by clicking its page index tab.
- Select **Select Effects** from the **Data** menu or click the **Select Effects** icon in the Control Panel.

The Effects window will appear, as shown next.



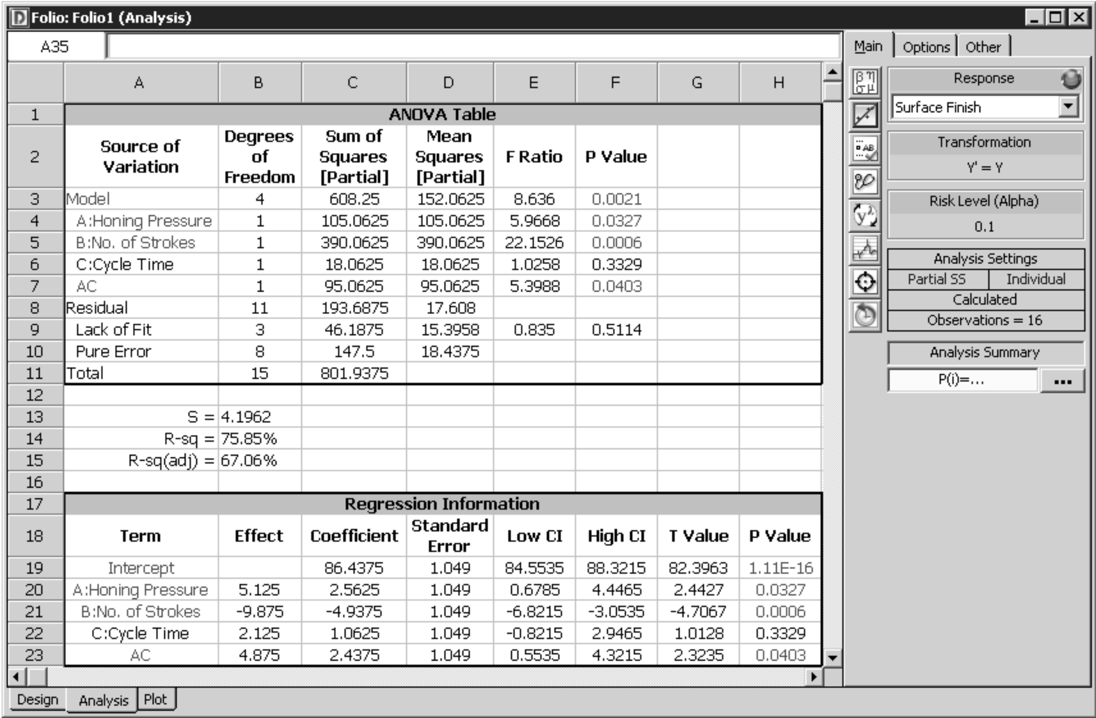
- Click the **Select Significant Effects** button to select only the effects that were shown to be significant. This will clear the checkboxes beside the AB, BC and ABC effects, leaving only effects A, B, C and AC selected, as shown next.




Note that selections in the Effects window must be hierarchical—if you select to include any second order or greater effect, all related main effects must also be included. Thus, in this case, effect C is included in order to include effect AC.

- Click **OK** to accept your selections and close the window. Note that the status indicator in the Response area of the Control Panel is now red, indicating that you have made changes since the data set was last analyzed, and the status in the information area of the Control Panel says "Edited."
- In the Analysis Settings area of the Control Panel, click inside the Test Terms area to toggle the analysis settings from **Grouped** to **Individual**. This performs the same function as changing from Grouped Terms to Individual Terms on the Options page of the Control Panel.

- Click the **Calculate** icon to re-analyze the data set using the selections you have just made. The changed Analysis tab is shown next.




| ANOVA Table | | | | | |
|---|---|---|---|---|---|
| Source of Variation | Degrees of Freedom | Sum of Squares [Partial] | Mean Squares [Partial] | F Ratio | P Value |
| Model | 4 | 608.25 | 152.0625 | 8.636 | 0.0021 |
| A:Honing Pressure | 1 | 105.0625 | 105.0625 | 5.9668 | 0.0327 |
| B:No. of Strokes | 1 | 390.0625 | 390.0625 | 22.1526 | 0.0006 |
| C:Cycle Time | 1 | 18.0625 | 18.0625 | 1.0258 | 0.3329 |
| AC | 1 | 95.0625 | 95.0625 | 5.3988 | 0.0403 |
| Residual | 11 | 193.6875 | 17.608 | | |
| Lack of Fit | 3 | 46.1875 | 15.3958 | 0.835 | 0.5114 |
| Pure Error | 8 | 147.5 | 18.4375 | | |
| Total | 15 | 801.9375 | | | |

S = 4.1962
R-sq = 75.85%
R-sq(adj) = 67.06%

| Regression Information | | | | | | | |
|---|---|---|---|---|---|---|---|
| Term | Effect | Coefficient | Standard Error | Low CI | High CI | T Value | P Value |
| Intercept | | 86.4375 | 1.049 | 84.5535 | 88.3215 | 82.3963 | 1.11E-16 |
| A:Honing Pressure | 5.125 | 2.5625 | 1.049 | 0.6785 | 4.4465 | 2.4427 | 0.0327 |
| B:No. of Strokes | -9.875 | -4.9375 | 1.049 | -6.8215 | -3.0535 | -4.7067 | 0.0006 |
| C:Cycle Time | 2.125 | 1.0625 | 1.049 | -0.8215 | 2.9465 | 1.0128 | 0.3329 |
| AC | 4.875 | 2.4375 | 1.049 | 0.5535 | 4.3215 | 2.3235 | 0.0403 |

- Click the **Show Analysis Summary** (**...**) icon at the lower right side of the Control Panel to view the analysis summary in the Results Panel. The Analysis Info page of the analysis summary is shown next.

Analysis Summary

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **Surface Finish Results** | | | | | |
| 2 | | | | | | |
| 3 | Transform: Y' = Y | | | | | |
| 4 | | | | | | |
| 5 | **Factor Properties** | | | | | |
| 6 | | | | | | |
| 7 | **Factor** | **Name** | **Units** | **Type** | **Low Level** | **High Level** |
| 8 | A | Honing Pressure | psi | Quantitative | 200 | 400 |
| 9 | B | No. of Strokes | | Quantitative | 3 | 5 |
| 10 | C | Cycle Time | sec | Quantitative | 3 | 5 |
| 11 | | | | | | |
| 12 | | | | | | |
| 13 | **Significant Effects at Alpha = 0.1:** | | | | | |
| 14 | | | | | | |
| 15 | | **Name** | **P Value** | | | |
| 16 | | A:Honing Pressure | 0.0327 | | | |
| 17 | | B:No. of Strokes | 0.0006 | | | |
| 18 | | AC | 0.0403 | | | |
| 19 | | | | | | |
| 20 | | | | | | |
| 21 | **Equation in terms of Coded values:** | | | | | |
| 22 | | | | | | |
| 23 | | 86.4375 | | | | |
| 24 | | +2.5625 | * A:Honing Pressure | | | |
| 25 | | -4.9375 | * B:No. of Strokes | | | |
| 26 | | +1.0625 | * C:Cycle Time | | | |
| 27 | | +2.4375 | * AC | | | |
| 28 | | | | | | |
| 29 | **Equation in terms of Actual values:** | | | | | |
| 30 | | | | | | |
| 31 | | 123.5 | | | | |
| 32 | | -0.0719 | * A:Honing Pressure | | | |
| 33 | | -4.9375 | * B:No. of Strokes | | | |
| 34 | | -6.2500 | * C:Cycle Time | | | |
| 35 | | +0.0244 | * AC | | | |

Analysis Info / Alias Structure / Var_Cov Matrix

The parameters of the model are shown in the **Equation in terms of Actual values** section. This can be rewritten as:

$$\hat{y} = 123.5000 - 0.0719x_1 - 4.9375x_2 - 6.2500x_3 + 0.0244x_1x_3$$

- Click the **Close** button in the Results Panel toolbar to close the window.
- Save the project by selecting **Save** from the **File** menu or by clicking the **Save** icon.
- Close the project by selecting **Close** from the **File** menu and proceed to the next example.

## 4.3 Example 3: Two Level Fractional Factorial Design

A researcher is interested in investigating different membrane operating conditions for an affinity membrane chromatographic column. The experiment consists of feeding a protein solution of concentration $C_o$ through the porous membrane column. As the solution runs through the column, targeted protein molecules are immobilized. The concentration of the effluent C is measured, which increases with time as the column approaches saturation. The response is the processing time, *i.e.* the time by which the effluent concentration reaches a breakthrough, $C_{BT}$, where $C_{BT} = C/C_o$. In other words, if $C_{BT}$ equals 0.1, the processing time is the time by which the concentration of the effluent reaches 10% of the concentration of the initial solution.

Given the large number of factors that the researcher is attempting to explore and due to time and resource constraints, she decides to perform a fractional factorial design with 32 runs with the following factor settings:

| Factor | Units | Low Level | High Level |
|---|---|---|---|
| Inlet concentration | $mol/m^3$ | 5E-5 | 10E-5 |
| Inlet flow-rate | $m^3/s$ | 1E-8 | 1E-7 |
| Column length | m | 0.5 | 1 |
| Column diameter | m | 0.01 | 0.1 |
| Bead porosity | - | 0.7 | 0.8 |
| Bead radius | m | 4E-5 | 8E-5 |
| CBT | - | 0.2 | 0.8 |

Results obtained from the experiment are shown next.

| **Inlet concentration ($mol/m^3$)** | **Inlet flow-rate ($m^3/s$)** | **Column length (m)** | **Column diameter (m)** | **Bead porosity** | **Bead radius (m)** | **CBT** | **Time (min)** |
|---|---|---|---|---|---|---|---|
| 5.00E-05 | 1.00E-08 | 0.5 | 0.01 | 0.7 | 0.0008 | 0.8 | 109 |
| 1.00E-04 | 1.00E-08 | 0.5 | 0.01 | 0.7 | 0.0004 | 0.2 | 100 |
| 5.00E-05 | 1.00E-07 | 0.5 | 0.01 | 0.7 | 0.0004 | 0.2 | 110 |
| 1.00E-04 | 1.00E-07 | 0.5 | 0.01 | 0.7 | 0.0008 | 0.8 | 105 |
| 5.00E-05 | 1.00E-08 | 1 | 0.01 | 0.7 | 0.0004 | 0.2 | 73 |
| 1.00E-04 | 1.00E-08 | 1 | 0.01 | 0.7 | 0.0008 | 0.8 | 111 |
| 5.00E-05 | 1.00E-07 | 1 | 0.01 | 0.7 | 0.0008 | 0.8 | 102 |
| 1.00E-04 | 1.00E-07 | 1 | 0.01 | 0.7 | 0.0004 | 0.2 | 73 |
| 5.00E-05 | 1.00E-08 | 0.5 | 0.1 | 0.7 | 0.0004 | 0.8 | 85 |
| 1.00E-04 | 1.00E-08 | 0.5 | 0.1 | 0.7 | 0.0008 | 0.2 | 98 |
| 5.00E-05 | 1.00E-07 | 0.5 | 0.1 | 0.7 | 0.0008 | 0.2 | 100 |
| 1.00E-04 | 1.00E-07 | 0.5 | 0.1 | 0.7 | 0.0004 | 0.8 | 83 |
| 5.00E-05 | 1.00E-08 | 1 | 0.1 | 0.7 | 0.0008 | 0.2 | 90 |
| 1.00E-04 | 1.00E-08 | 1 | 0.1 | 0.7 | 0.0004 | 0.8 | 93 |
| 5.00E-05 | 1.00E-07 | 1 | 0.1 | 0.7 | 0.0004 | 0.8 | 86 |
| 1.00E-04 | 1.00E-07 | 1 | 0.1 | 0.7 | 0.0008 | 0.2 | 84 |

| Inlet concentration (mol/m$^3$) | Inlet flow-rate (m$^3$/s) | Column length (m) | Column diameter (m) | Bead porosity | Bead radius (m) | CBT | Time (min) |
|---|---|---|---|---|---|---|---|
| 5.00E-05 | 1.00E-08 | 0.5 | 0.01 | 0.8 | 0.0008 | 0.2 | 47 |
| 1.00E-04 | 1.00E-08 | 0.5 | 0.01 | 0.8 | 0.0004 | 0.8 | 155 |
| 5.00E-05 | 1.00E-07 | 0.5 | 0.01 | 0.8 | 0.0004 | 0.8 | 165 |
| 1.00E-04 | 1.00E-07 | 0.5 | 0.01 | 0.8 | 0.0008 | 0.2 | 50 |
| 5.00E-05 | 1.00E-08 | 1 | 0.01 | 0.8 | 0.0004 | 0.8 | 153 |
| 1.00E-04 | 1.00E-08 | 1 | 0.01 | 0.8 | 0.0008 | 0.2 | 30 |
| 5.00E-05 | 1.00E-07 | 1 | 0.01 | 0.8 | 0.0008 | 0.2 | 33 |
| 1.00E-04 | 1.00E-07 | 1 | 0.01 | 0.8 | 0.0004 | 0.8 | 150 |
| 5.00E-05 | 1.00E-08 | 0.5 | 0.1 | 0.8 | 0.0004 | 0.2 | 44 |
| 1.00E-04 | 1.00E-08 | 0.5 | 0.1 | 0.8 | 0.0008 | 0.8 | 142 |
| 5.00E-05 | 1.00E-07 | 0.5 | 0.1 | 0.8 | 0.0008 | 0.8 | 139 |
| 1.00E-04 | 1.00E-07 | 0.5 | 0.1 | 0.8 | 0.0004 | 0.2 | 40 |
| 5.00E-05 | 1.00E-08 | 1 | 0.1 | 0.8 | 0.0008 | 0.8 | 148 |
| 1.00E-04 | 1.00E-08 | 1 | 0.1 | 0.8 | 0.0004 | 0.2 | 21 |
| 5.00E-05 | 1.00E-07 | 1 | 0.1 | 0.8 | 0.0004 | 0.2 | 23 |
| 1.00E-04 | 1.00E-07 | 1 | 0.1 | 0.8 | 0.0008 | 0.8 | 151 |

Do the following:

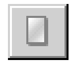
- Examine the fraction generators and the alias structure.
- Identify the significant effects using the Effect Probability plot and reduce the model accordingly.[5]
- If the researcher is to determine membrane operating conditions that will optimize the column capacity (*i.e.* minimize the breakthrough time) what should the next step be?

*The sample file for this example is located in the "Training Guide" folder in your application directory (e.g. C:\Program Files\ReliaSoft\DOE\Training Guide) and is named "TwoLevelFractionalFactorial.rdoe." The data set for this example is available in the Spreadsheet in the "TwoLevelFractionalFactorial-Data.rso7" file located in the "Raw Data" folder within the "Training Guide" folder.*

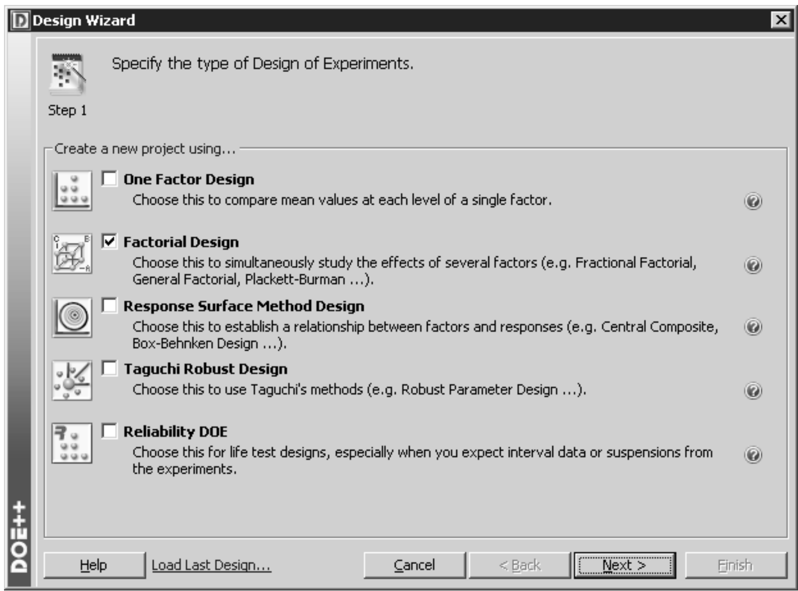
5. Note that since there are no replicates in this design, Lenth's method is applied in the Effect Probability plot to determine significant effects. Alternatively, the analyst might choose to assume the third order effects are negligible.

**Solution**

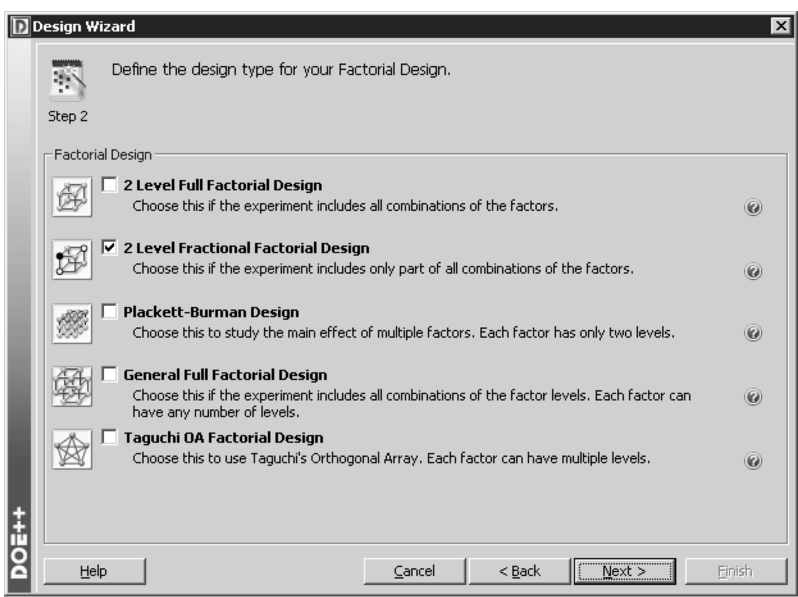
- Create a new project by clicking **Create a New Project** in the initial window, by selecting **New** from the **File** menu or by clicking the **New** icon.



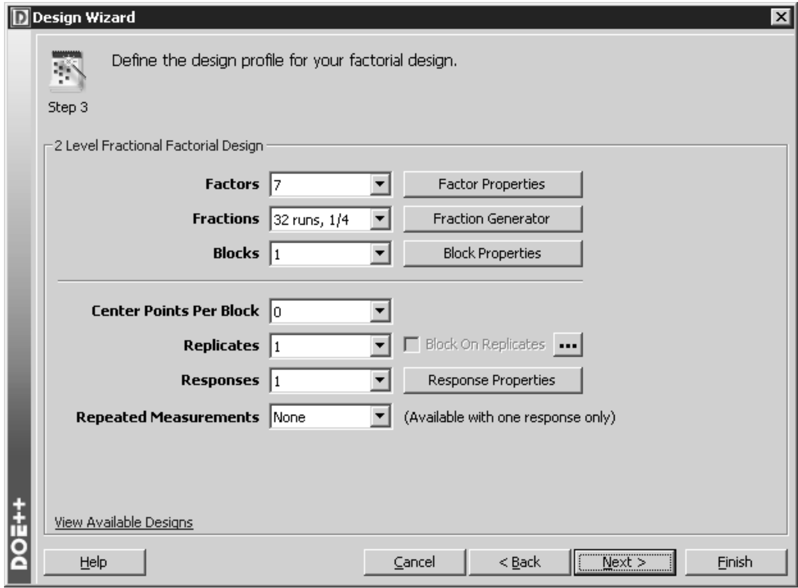
- On the first page of the Design Wizard, select to create a **Factorial Design** and click **Next** to proceed to the next step.



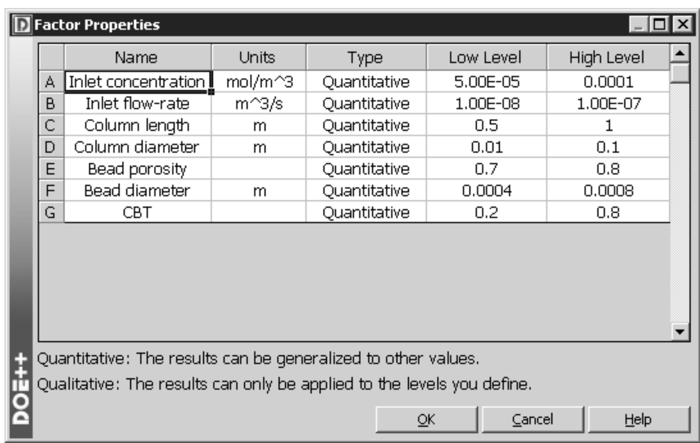

- In the second step of the Design Wizard, select **2 Level Fractional Factorial Design** and click **Next** to proceed to the next step.



- In the third step of the Design Wizard, select **7** from the **Factors** drop-down and **32 runs, 1/4** from the **Fractions** drop-down and leave all other fields at their default settings, as shown next.



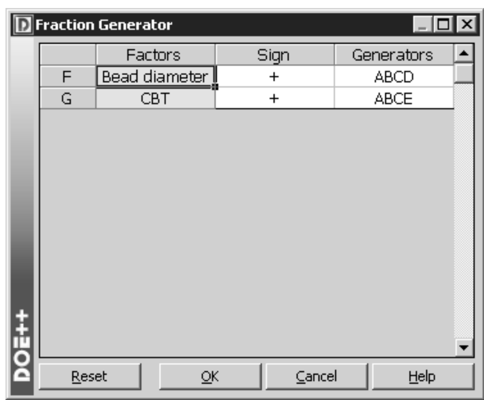

- Click the **Factor Properties** button. In the Factor Properties window that appears, enter the names, units, types and high and low level values for the factors, as shown next.




- Click **OK** to close the Factor Properties window and return to the Design Wizard.

- Click the **Response Properties** button. In the Response Properties window that appears, define the response name, as shown next.

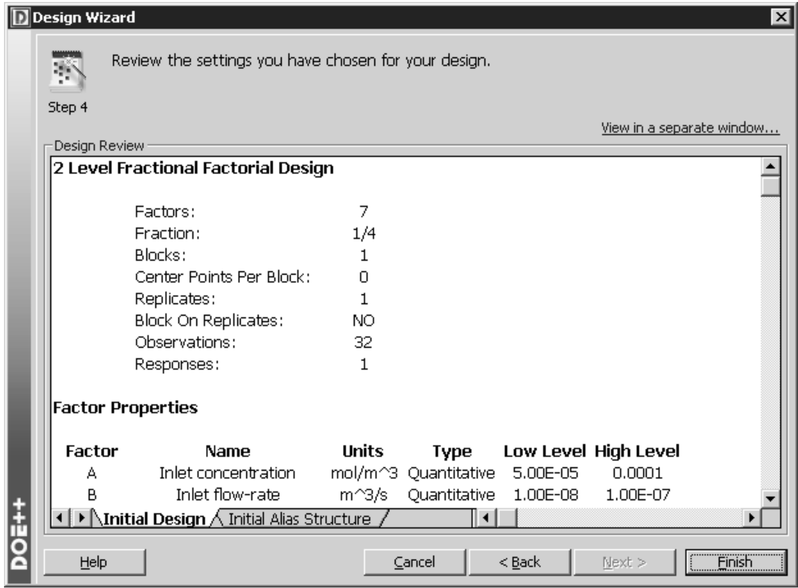

- Click **OK** to close the Response Properties window and return to the Design Wizard.
- Click the **Fraction Generator** button to open the Fraction Generator window, which allows you to specify the generators, or factorial interactions whose effects will be aliased with the main effects of particular factors. Default generators are automatically supplied, as shown next.



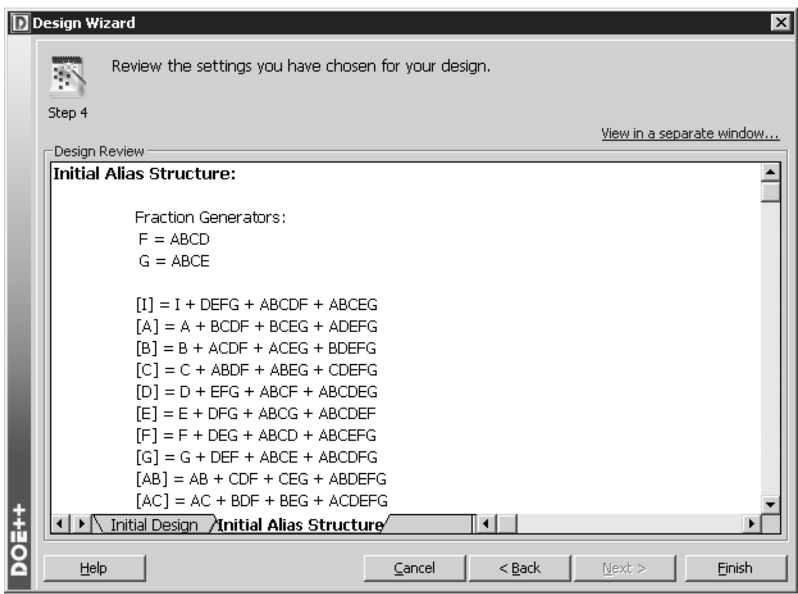

These generators create an alias structure for the design, which we will examine in an upcoming step.

- Accept the default generators and click **OK** to close the window and return to the Design Wizard.
- Click **Next** to proceed to the next step.

- The fourth step of the Design Wizard provides a review of the settings you have chosen for the design.




Note that there is an additional tab called "Initial Alias Structure" in the summary area. Click this tab to view the alias structure of the design, as shown next.

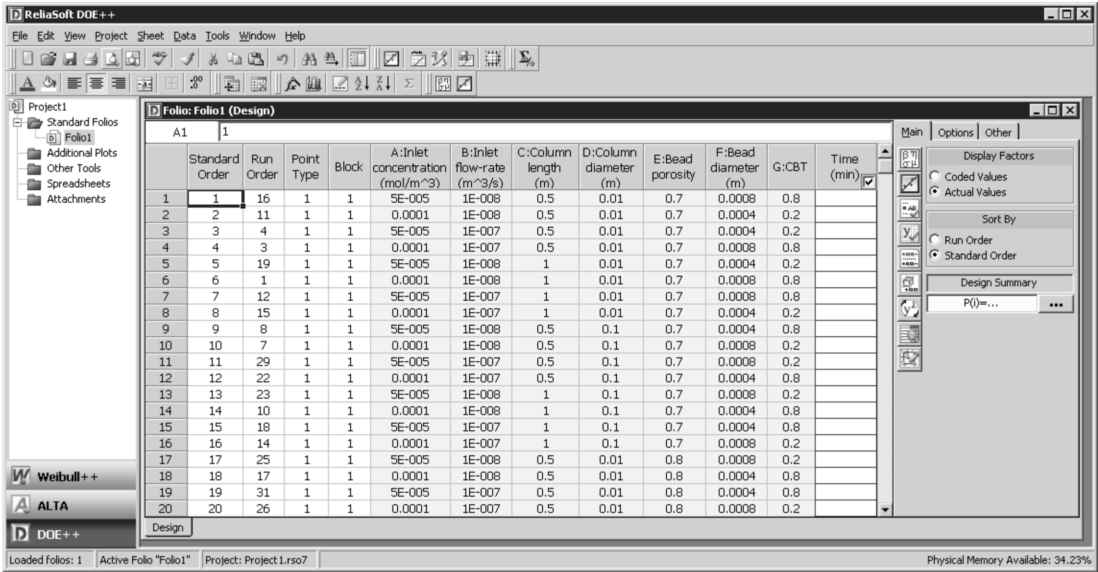



If any of the settings shown in these two tabs are not as intended, you can click the **Back** button to return to the appropriate step and make any necessary changes.

- Click **Finish** to create the new project with the Standard Folio containing the two level fractional factorial design.

By default, the Design tab Data Sheet is sorted by the Run Order column, which is the randomized order, generated by DOE++, in which it is recommended to perform the runs to avoid biased results. For ease of entry, the data in this example have been presented in the standard order as specified in the design type.

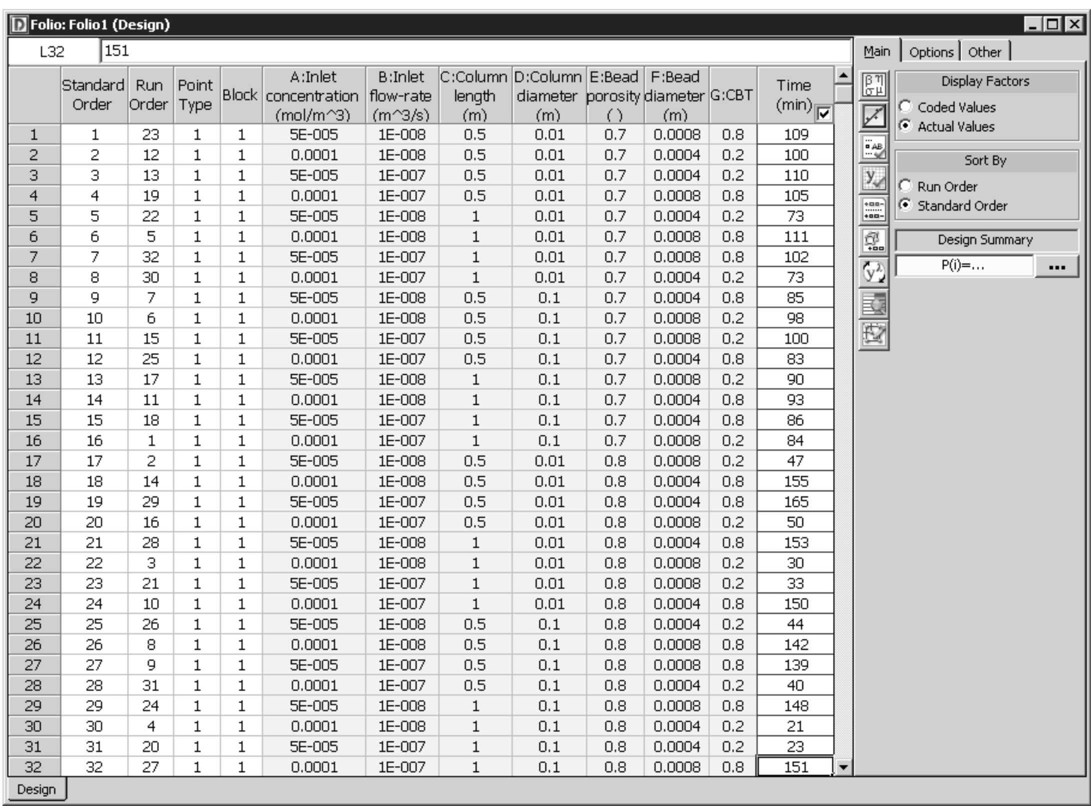
- To sort the Data Sheet by the Standard Order column, select **Standard Order** in the **Sort By** area in the Control Panel. The Standard Folio will look like the one shown next, as shown next.



| | Standard Order | Run Order | Point Type | Block | A:Inlet concentration (mol/m^3) | B:Inlet flow-rate (m^3/s) | C:Column length (m) | D:Column diameter (m) | E:Bead porosity | F:Bead diameter (m) | G:CBT | Time (min) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 16 | 1 | 1 | 5E-005 | 1E-008 | 0.5 | 0.01 | 0.7 | 0.0008 | 0.8 | |
| 2 | 2 | 11 | 1 | 1 | 0.0001 | 1E-008 | 0.5 | 0.01 | 0.7 | 0.0004 | 0.2 | |
| 3 | 3 | 4 | 1 | 1 | 5E-005 | 1E-007 | 0.5 | 0.01 | 0.7 | 0.0004 | 0.2 | |
| 4 | 4 | 3 | 1 | 1 | 0.0001 | 1E-007 | 0.5 | 0.01 | 0.7 | 0.0008 | 0.8 | |
| 5 | 5 | 19 | 1 | 1 | 5E-005 | 1E-008 | 1 | 0.01 | 0.7 | 0.0004 | 0.2 | |
| 6 | 6 | 1 | 1 | 1 | 0.0001 | 1E-008 | 1 | 0.01 | 0.7 | 0.0008 | 0.8 | |
| 7 | 7 | 12 | 1 | 1 | 5E-005 | 1E-007 | 1 | 0.01 | 0.7 | 0.0008 | 0.8 | |
| 8 | 8 | 15 | 1 | 1 | 0.0001 | 1E-007 | 1 | 0.01 | 0.7 | 0.0004 | 0.2 | |
| 9 | 9 | 8 | 1 | 1 | 5E-005 | 1E-008 | 0.5 | 0.1 | 0.7 | 0.0004 | 0.8 | |
| 10 | 10 | 7 | 1 | 1 | 0.0001 | 1E-008 | 0.5 | 0.1 | 0.7 | 0.0008 | 0.2 | |
| 11 | 11 | 29 | 1 | 1 | 5E-005 | 1E-007 | 0.5 | 0.1 | 0.7 | 0.0008 | 0.2 | |
| 12 | 12 | 22 | 1 | 1 | 0.0001 | 1E-007 | 0.5 | 0.1 | 0.7 | 0.0004 | 0.8 | |
| 13 | 13 | 23 | 1 | 1 | 5E-005 | 1E-008 | 1 | 0.1 | 0.7 | 0.0008 | 0.2 | |
| 14 | 14 | 10 | 1 | 1 | 0.0001 | 1E-008 | 1 | 0.1 | 0.7 | 0.0004 | 0.8 | |
| 15 | 15 | 18 | 1 | 1 | 5E-005 | 1E-007 | 1 | 0.1 | 0.7 | 0.0004 | 0.8 | |
| 16 | 16 | 14 | 1 | 1 | 0.0001 | 1E-007 | 1 | 0.1 | 0.7 | 0.0008 | 0.2 | |
| 17 | 17 | 25 | 1 | 1 | 5E-005 | 1E-008 | 0.5 | 0.01 | 0.8 | 0.0008 | 0.2 | |
| 18 | 18 | 17 | 1 | 1 | 0.0001 | 1E-008 | 0.5 | 0.01 | 0.8 | 0.0004 | 0.8 | |
| 19 | 19 | 31 | 1 | 1 | 5E-005 | 1E-007 | 0.5 | 0.01 | 0.8 | 0.0004 | 0.8 | |
| 20 | 20 | 26 | 1 | 1 | 0.0001 | 1E-007 | 0.5 | 0.01 | 0.8 | 0.0008 | 0.2 | |

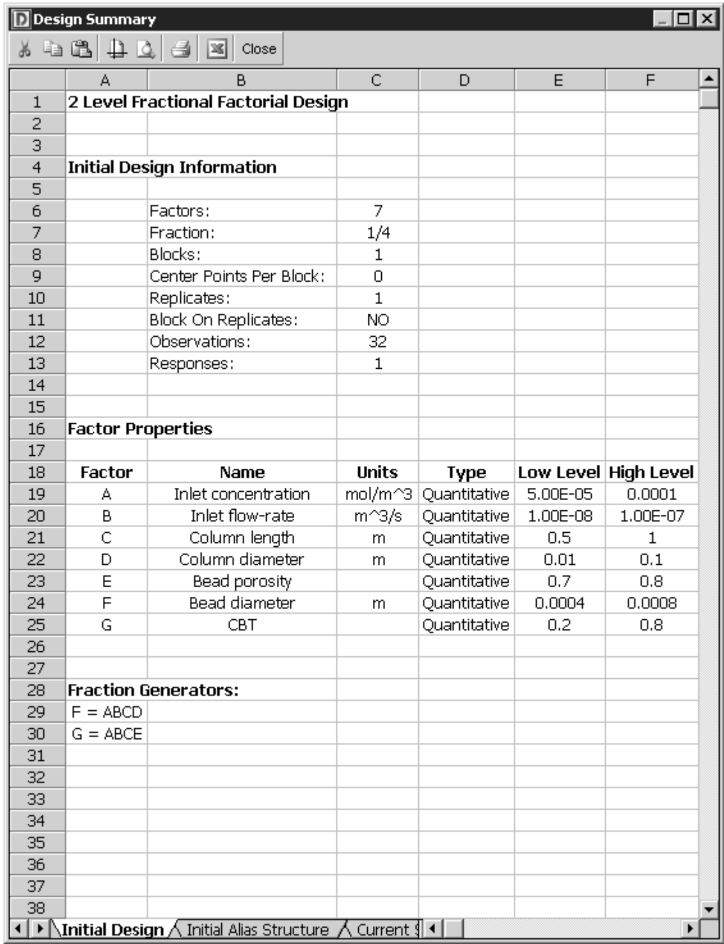
Note that not all rows are displayed in the image above. Your design will contain 32 rows.

- Save the project as "TwoLevelFractionalFactorial.rso7."
- Enter the response data in the order given in this example on page 37. The Folio will look like the one shown next.



| | Standard Order | Run Order | Point Type | Block | A:Inlet concentration (mol/m^3) | B:Inlet flow-rate (m^3/s) | C:Column length (m) | D:Column diameter (m) | E:Bead porosity ( ) | F:Bead diameter (m) | G:CBT | Time (min) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 23 | 1 | 1 | 5E-005 | 1E-008 | 0.5 | 0.01 | 0.7 | 0.0008 | 0.8 | 109 |
| 2 | 2 | 12 | 1 | 1 | 0.0001 | 1E-008 | 0.5 | 0.01 | 0.7 | 0.0004 | 0.2 | 100 |
| 3 | 3 | 13 | 1 | 1 | 5E-005 | 1E-007 | 0.5 | 0.01 | 0.7 | 0.0004 | 0.2 | 110 |
| 4 | 4 | 19 | 1 | 1 | 0.0001 | 1E-007 | 0.5 | 0.01 | 0.7 | 0.0008 | 0.8 | 105 |
| 5 | 5 | 22 | 1 | 1 | 5E-005 | 1E-008 | 1 | 0.01 | 0.7 | 0.0004 | 0.2 | 73 |
| 6 | 6 | 5 | 1 | 1 | 0.0001 | 1E-008 | 1 | 0.01 | 0.7 | 0.0008 | 0.8 | 111 |
| 7 | 7 | 32 | 1 | 1 | 5E-005 | 1E-007 | 1 | 0.01 | 0.7 | 0.0008 | 0.8 | 102 |
| 8 | 8 | 30 | 1 | 1 | 0.0001 | 1E-007 | 1 | 0.01 | 0.7 | 0.0004 | 0.2 | 73 |
| 9 | 9 | 7 | 1 | 1 | 5E-005 | 1E-008 | 0.5 | 0.1 | 0.7 | 0.0004 | 0.8 | 85 |
| 10 | 10 | 6 | 1 | 1 | 0.0001 | 1E-008 | 0.5 | 0.1 | 0.7 | 0.0008 | 0.2 | 98 |
| 11 | 11 | 15 | 1 | 1 | 5E-005 | 1E-007 | 0.5 | 0.1 | 0.7 | 0.0008 | 0.2 | 100 |
| 12 | 12 | 25 | 1 | 1 | 0.0001 | 1E-007 | 0.5 | 0.1 | 0.7 | 0.0004 | 0.8 | 83 |
| 13 | 13 | 17 | 1 | 1 | 5E-005 | 1E-008 | 1 | 0.1 | 0.7 | 0.0008 | 0.2 | 90 |
| 14 | 14 | 11 | 1 | 1 | 0.0001 | 1E-008 | 1 | 0.1 | 0.7 | 0.0004 | 0.8 | 93 |
| 15 | 15 | 18 | 1 | 1 | 5E-005 | 1E-007 | 1 | 0.1 | 0.7 | 0.0004 | 0.8 | 86 |
| 16 | 16 | 1 | 1 | 1 | 0.0001 | 1E-007 | 1 | 0.1 | 0.7 | 0.0008 | 0.2 | 84 |
| 17 | 17 | 2 | 1 | 1 | 5E-005 | 1E-008 | 0.5 | 0.01 | 0.8 | 0.0008 | 0.2 | 47 |
| 18 | 18 | 14 | 1 | 1 | 0.0001 | 1E-008 | 0.5 | 0.01 | 0.8 | 0.0004 | 0.8 | 155 |
| 19 | 19 | 29 | 1 | 1 | 5E-005 | 1E-007 | 0.5 | 0.01 | 0.8 | 0.0004 | 0.8 | 165 |
| 20 | 20 | 16 | 1 | 1 | 0.0001 | 1E-007 | 0.5 | 0.01 | 0.8 | 0.0008 | 0.2 | 50 |
| 21 | 21 | 28 | 1 | 1 | 5E-005 | 1E-008 | 1 | 0.01 | 0.8 | 0.0004 | 0.8 | 153 |
| 22 | 22 | 3 | 1 | 1 | 0.0001 | 1E-008 | 1 | 0.01 | 0.8 | 0.0008 | 0.2 | 30 |
| 23 | 23 | 21 | 1 | 1 | 5E-005 | 1E-007 | 1 | 0.01 | 0.8 | 0.0008 | 0.2 | 33 |
| 24 | 24 | 10 | 1 | 1 | 0.0001 | 1E-007 | 1 | 0.01 | 0.8 | 0.0004 | 0.8 | 150 |
| 25 | 25 | 26 | 1 | 1 | 5E-005 | 1E-008 | 0.5 | 0.1 | 0.8 | 0.0004 | 0.2 | 44 |
| 26 | 26 | 8 | 1 | 1 | 0.0001 | 1E-008 | 0.5 | 0.1 | 0.8 | 0.0008 | 0.8 | 142 |
| 27 | 27 | 9 | 1 | 1 | 5E-005 | 1E-007 | 0.5 | 0.1 | 0.8 | 0.0008 | 0.8 | 139 |
| 28 | 28 | 31 | 1 | 1 | 0.0001 | 1E-007 | 0.5 | 0.1 | 0.8 | 0.0004 | 0.2 | 40 |
| 29 | 29 | 24 | 1 | 1 | 5E-005 | 1E-008 | 1 | 0.1 | 0.8 | 0.0008 | 0.8 | 148 |
| 30 | 30 | 4 | 1 | 1 | 0.0001 | 1E-008 | 1 | 0.1 | 0.8 | 0.0004 | 0.2 | 21 |
| 31 | 31 | 20 | 1 | 1 | 5E-005 | 1E-007 | 1 | 0.1 | 0.8 | 0.0004 | 0.2 | 23 |
| 32 | 32 | 27 | 1 | 1 | 0.0001 | 1E-007 | 1 | 0.1 | 0.8 | 0.0008 | 0.8 | 151 |

- To examine the alias structure for the design, click the **Show Design Summary** (**...**) icon at the lower right side of the Control Panel. The design summary will be displayed in the Results Panel, as shown next.

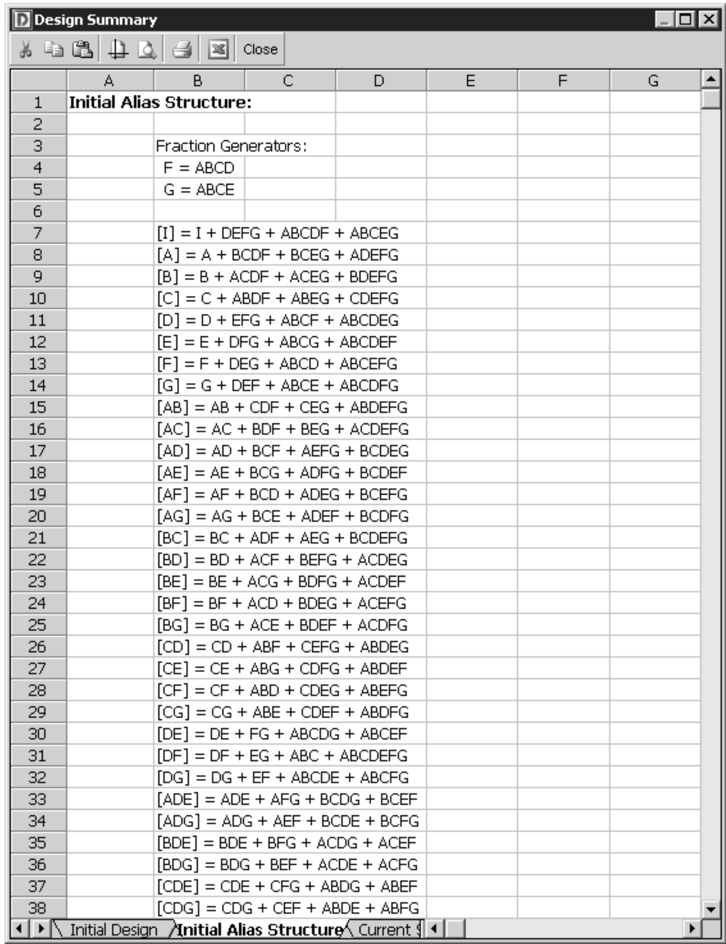

Design Summary

Close

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **2 Level Fractional Factorial Design** | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | **Initial Design Information** | | | | | |
| 5 | | | | | | |
| 6 | | Factors: | 7 | | | |
| 7 | | Fraction: | 1/4 | | | |
| 8 | | Blocks: | 1 | | | |
| 9 | | Center Points Per Block: | 0 | | | |
| 10 | | Replicates: | 1 | | | |
| 11 | | Block On Replicates: | NO | | | |
| 12 | | Observations: | 32 | | | |
| 13 | | Responses: | 1 | | | |
| 14 | | | | | | |
| 15 | | | | | | |
| 16 | **Factor Properties** | | | | | |
| 17 | | | | | | |
| 18 | **Factor** | **Name** | **Units** | **Type** | **Low Level** | **High Level** |
| 19 | A | Inlet concentration | mol/m^3 | Quantitative | 5.00E-05 | 0.0001 |
| 20 | B | Inlet flow-rate | m^3/s | Quantitative | 1.00E-08 | 1.00E-07 |
| 21 | C | Column length | m | Quantitative | 0.5 | 1 |
| 22 | D | Column diameter | m | Quantitative | 0.01 | 0.1 |
| 23 | E | Bead porosity | | Quantitative | 0.7 | 0.8 |
| 24 | F | Bead diameter | m | Quantitative | 0.0004 | 0.0008 |
| 25 | G | CBT | | Quantitative | 0.2 | 0.8 |
| 26 | | | | | | |
| 27 | | | | | | |
| 28 | **Fraction Generators:** | | | | | |
| 29 | F = ABCD | | | | | |
| 30 | G = ABCE | | | | | |
| 31 | | | | | | |
| 32 | | | | | | |
| 33 | | | | | | |
| 34 | | | | | | |
| 35 | | | | | | |
| 36 | | | | | | |
| 37 | | | | | | |
| 38 | | | | | | |

Initial Design / Initial Alias Structure / Current S

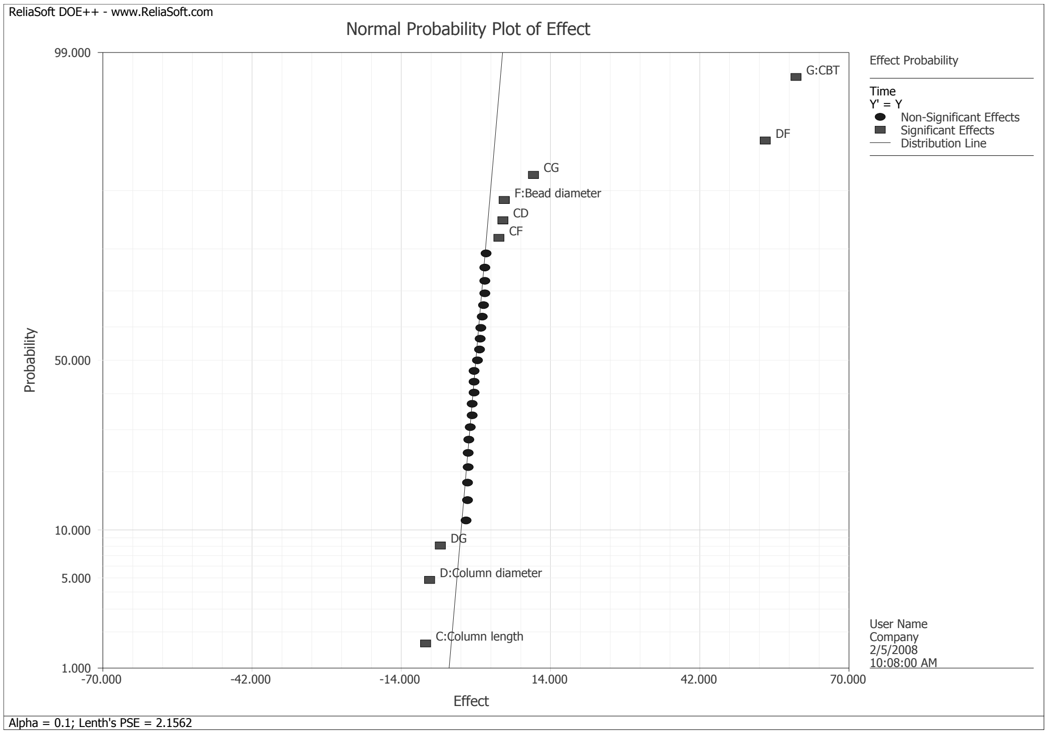
Click the Alias Structure page index tab of the design summary to view the initial alias structure, as shown next.




This specifies every instance of aliasing (or confounding) in the design, where the effect of a certain factor or factorial interaction cannot be separated from another effect.

- Click **Close** to close the Results Panel.
- On the Options page of the Control Panel, specify a **Risk Level (Alpha)** value of **0.1** for the Time response and select to use **Grouped Terms**.
- Return to the Main page of the Control Panel and click the **Plot** icon.

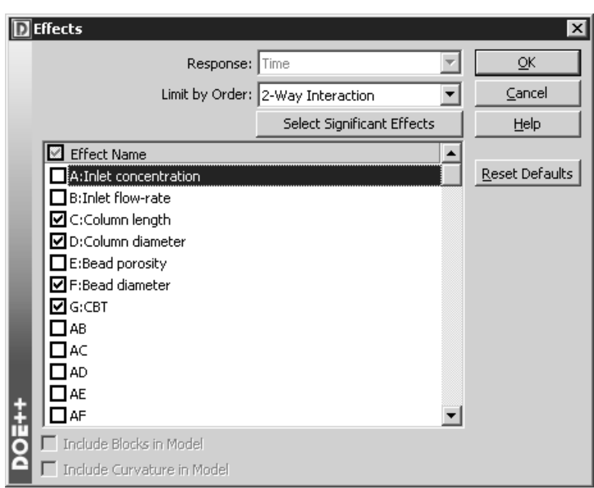
The data set will be analyzed automatically and the Analysis and Plot tabs will be added to the Folio. The Effect Probability plot, created by default, is shown next.



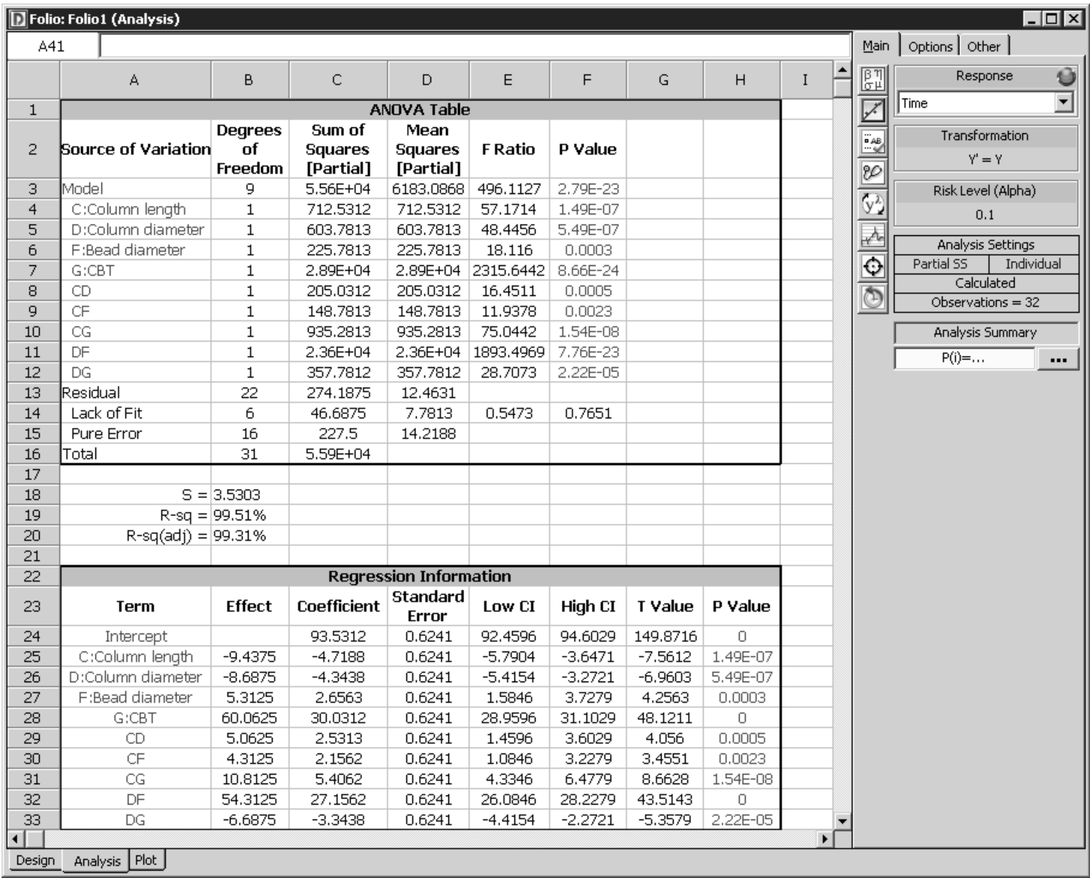

The significant main effects are shown to be C (column length), D (column diameter), F (bead diameter) and G (CBT). Significant interactions are CD, CF, CG, DF and DG.

- Go to the Analysis tab by clicking its page index tab.
- Select **Select Effects** from the **Data** menu or click the **Select Effects** icon in the Control Panel.

- In the Effects window, click the **Select Significant Effects** button to select only the effects shown to be significant (C, D, F, G, CD, CF, CG, DF and DG), as shown next. Note that not all selections are displayed in this image.



- Click **OK** to accept your selections and close the window.
- In the Analysis Settings area of the Control Panel, click inside the Test Terms area to toggle the analysis settings from **Grouped** to **Individual**.
- Click the **Calculate** icon to re-analyze the data set using the selections you have just made. The changed Analysis tab is shown next.




| ANOVA Table | | | | | |
|---|---|---|---|---|---|
| Source of Variation | Degrees of Freedom | Sum of Squares [Partial] | Mean Squares [Partial] | F Ratio | P Value |
| Model | 9 | 5.56E+04 | 6183.0868 | 496.1127 | 2.79E-23 |
| C:Column length | 1 | 712.5312 | 712.5312 | 57.1714 | 1.49E-07 |
| D:Column diameter | 1 | 603.7813 | 603.7813 | 48.4456 | 5.49E-07 |
| F:Bead diameter | 1 | 225.7813 | 225.7813 | 18.116 | 0.0003 |
| G:CBT | 1 | 2.89E+04 | 2.89E+04 | 2315.6442 | 8.66E-24 |
| CD | 1 | 205.0312 | 205.0312 | 16.4511 | 0.0005 |
| CF | 1 | 148.7813 | 148.7813 | 11.9378 | 0.0023 |
| CG | 1 | 935.2813 | 935.2813 | 75.0442 | 1.54E-08 |
| DF | 1 | 2.36E+04 | 2.36E+04 | 1893.4969 | 7.76E-23 |
| DG | 1 | 357.7812 | 357.7812 | 28.7073 | 2.22E-05 |
| Residual | 22 | 274.1875 | 12.4631 | | |
| Lack of Fit | 6 | 46.6875 | 7.7813 | 0.5473 | 0.7651 |
| Pure Error | 16 | 227.5 | 14.2188 | | |
| Total | 31 | 5.59E+04 | | | |

S = 3.5303
R-sq = 99.51%
R-sq(adj) = 99.31%

| Regression Information | | | | | | | |
|---|---|---|---|---|---|---|---|
| Term | Effect | Coefficient | Standard Error | Low CI | High CI | T Value | P Value |
| Intercept | | 93.5312 | 0.6241 | 92.4596 | 94.6029 | 149.8716 | 0 |
| C:Column length | -9.4375 | -4.7188 | 0.6241 | -5.7904 | -3.6471 | -7.5612 | 1.49E-07 |
| D:Column diameter | -8.6875 | -4.3438 | 0.6241 | -5.4154 | -3.2721 | -6.9603 | 5.49E-07 |
| F:Bead diameter | 5.3125 | 2.6563 | 0.6241 | 1.5846 | 3.7279 | 4.2563 | 0.0003 |
| G:CBT | 60.0625 | 30.0312 | 0.6241 | 28.9596 | 31.1029 | 48.1211 | 0 |
| CD | 5.0625 | 2.5313 | 0.6241 | 1.4596 | 3.6029 | 4.056 | 0.0005 |
| CF | 4.3125 | 2.1562 | 0.6241 | 1.0846 | 3.2279 | 3.4551 | 0.0023 |
| CG | 10.8125 | 5.4062 | 0.6241 | 4.3346 | 6.4779 | 8.6628 | 1.54E-08 |
| DF | 54.3125 | 27.1562 | 0.6241 | 26.0846 | 28.2279 | 43.5143 | 0 |
| DG | -6.6875 | -3.3438 | 0.6241 | -4.4154 | -2.2721 | -5.3579 | 2.22E-05 |

If the researcher is to determine membrane operating conditions that will optimize the column capacity (*i.e.* minimize the breakthrough time), she might chose to augment the design by adding center points. If the curvature is found to be significant, then a response surface methodology analysis might be beneficial. Otherwise, optimization with the current design can be performed. These steps are demonstrated in "Example 7: Sequential Optimization" on page 70.

- Save and close the project then proceed to the next example.

## 4.4 Example 4: General Full Factorial Design

During warranty analysis, a cylinder assembly was found to be problematic, translating into more warranty returns than expected. As part of the analysis, the engineers would like to know if different factors have a significant effect on the high number of warranty claims observed. Factors that they would like to explore include the region where the assembly was deployed (Region A, B or C), the engine type (gasoline, diesel, ethanol or hybrid) and the supplier for key components (S1 or S2).

The analyzed response was measured in terms of the reliability at the end of the warranty period and was obtained through warranty analysis of sold items and warranty claims. Note that the results were not obtained through a controlled experiment, as the warranty claims were recorded as they arrived.

The data set is shown next.

| Region | Engine Type | Supplier | Reliability at Warranty |
|---|---|---|---|
| A | Gasoline | S1 | 0.973 |
| A | Gasoline | S2 | 0.991 |
| A | Diesel | S1 | 0.971 |
| A | Diesel | S2 | 0.971 |
| A | Hybrid | S1 | 0.978 |
| A | Hybrid | S2 | 0.956 |
| A | Ethanol | S1 | 0.988 |
| A | Ethanol | S2 | 0.978 |
| B | Gasoline | S1 | 0.988 |
| B | Gasoline | S2 | 0.978 |
| B | Diesel | S1 | 0.992 |
| B | Diesel | S2 | 0.976 |
| B | Hybrid | S1 | 0.999 |
| B | Hybrid | S2 | 0.977 |
| B | Ethanol | S1 | 0.995 |
| B | Ethanol | S2 | 0.975 |
| C | Gasoline | S1 | 0.977 |
| C | Gasoline | S2 | 0.93 |
| C | Diesel | S1 | 0.971 |
| C | Diesel | S2 | 0.928 |
| C | Hybrid | S1 | 0.96 |
| C | Hybrid | S2 | 0.896 |
| C | Ethanol | S1 | 0.916 |
| C | Ethanol | S2 | 0.932 |

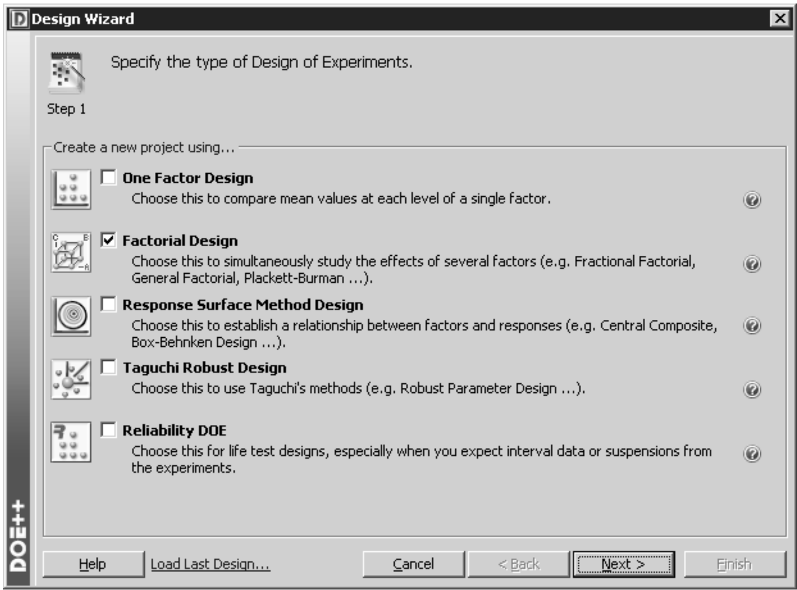
Do the following:

- Identify the significant effects, assuming that third order effects are negligible.
- Identify the critical population(s).

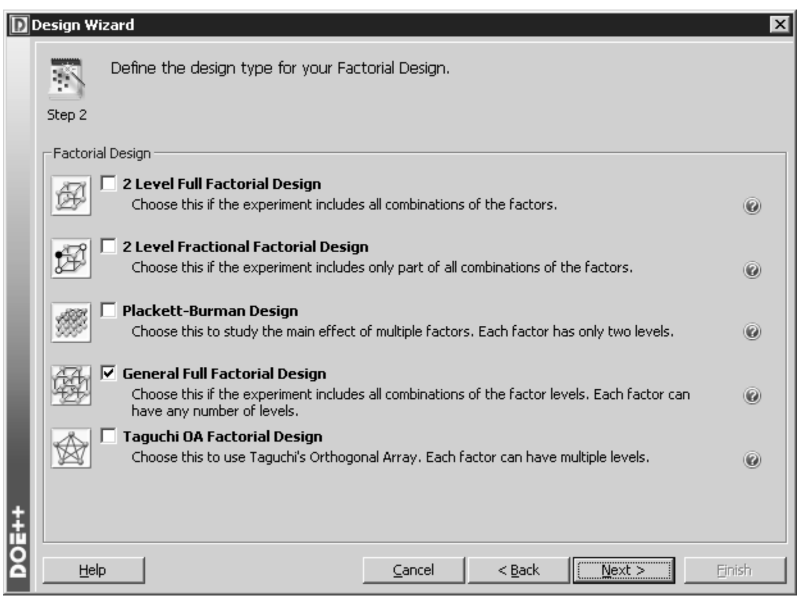
*The sample file for this example is located in the "Training Guide" folder in your application directory (e.g. C:\Program Files\ReliaSoft\DOE\Training Guide) and is named "GeneralFullFactorial.rdoe." The data set for this example is available in the Spreadsheet in the "GeneralFullFactorial-Data.rso7" file located in the "Raw Data" folder within the "Training Guide" folder.*

**Solution**

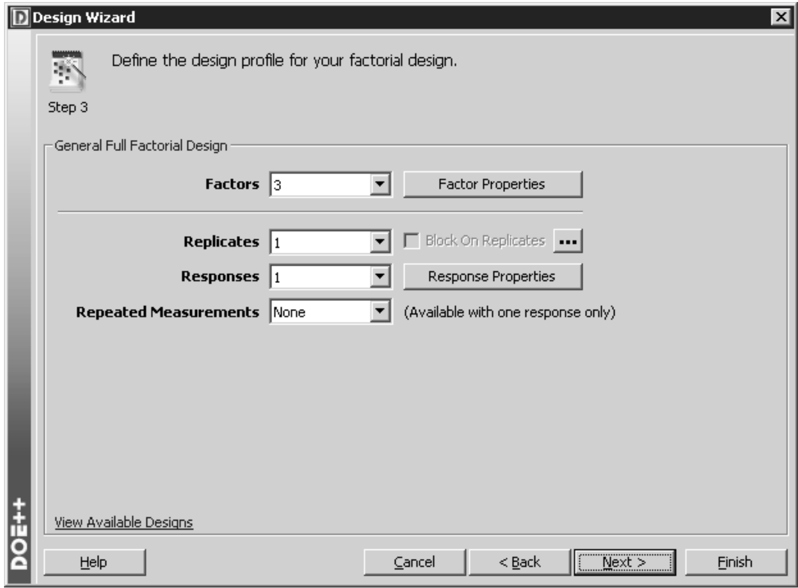
- Create a new project. On the first page of the Design Wizard, select to create a **Factorial Design** and click **Next** to proceed to the next step.



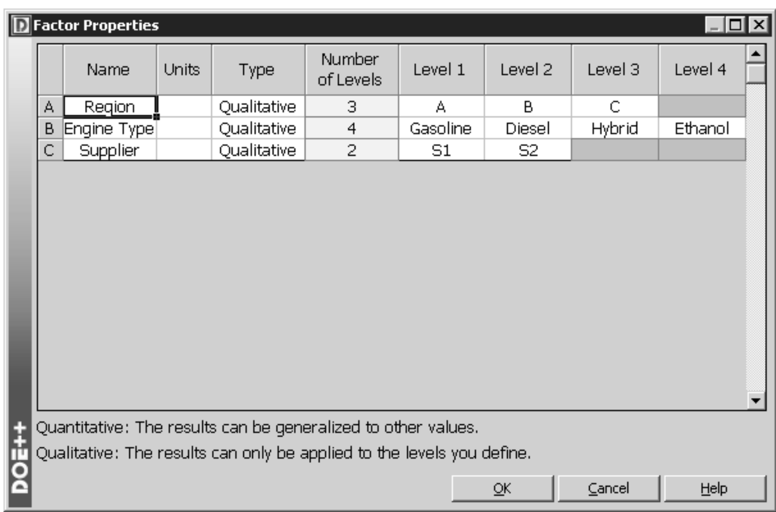

- In the second step of the Design Wizard, select **General Full Factorial Design** and click **Next** to proceed to the next step.



- In the third step of the Design Wizard, select **3** from the **Factors** drop-down and leave all other fields at their default settings, as shown next.

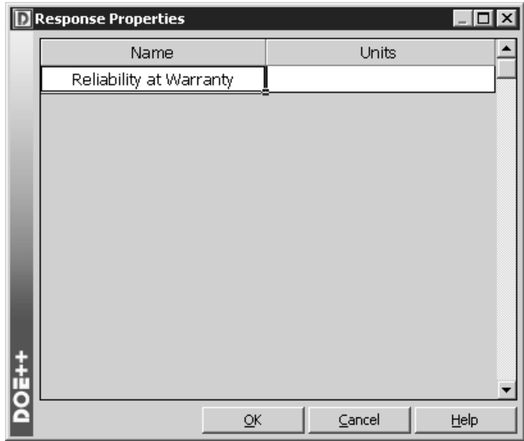



- Click the **Factor Properties** button. In the Factor Properties window that appears, enter the names and units for the factors then specify the number of levels for each factor and enter the level values, as shown next.

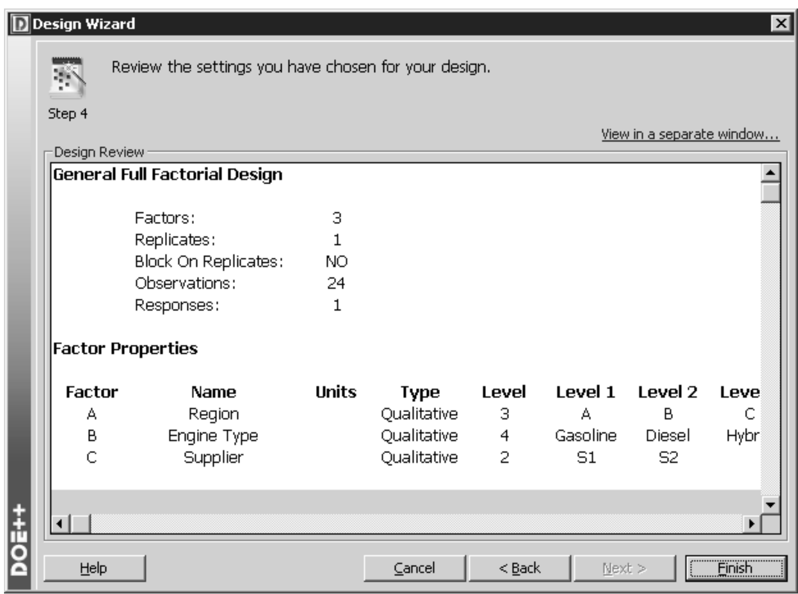



- Click **OK** to close the Factor Properties window and return to the Design Wizard.

- Click the **Response Properties** button. In the Response Properties window that appears, define the response name, as shown next.



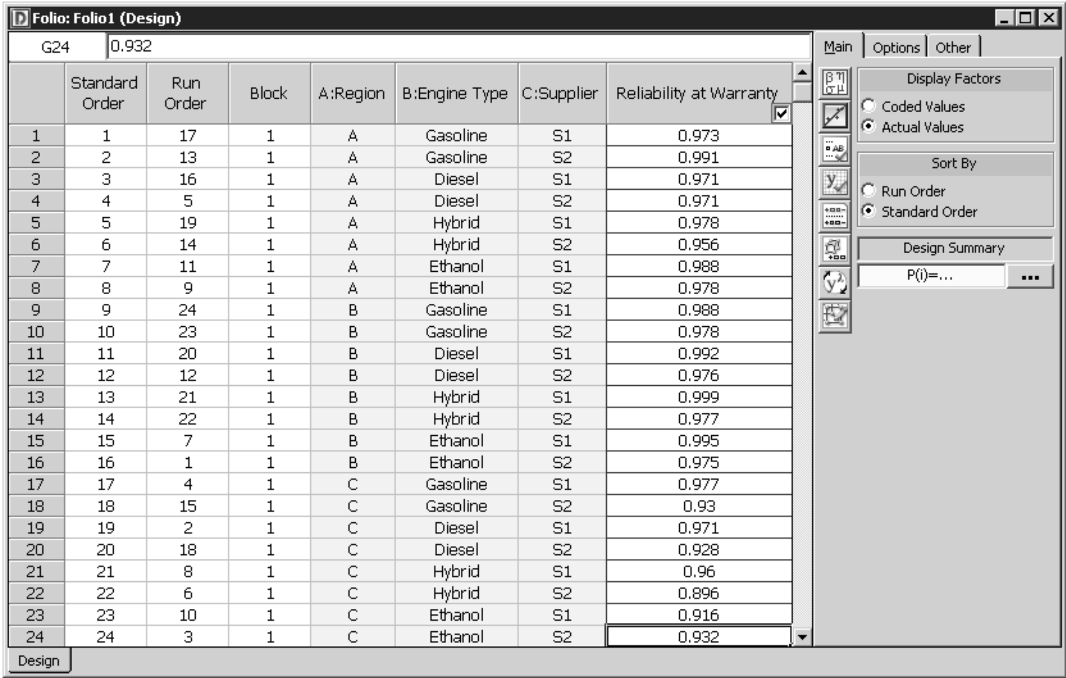

- Click **OK** to close the Response Properties window and return to the Design Wizard.
- Click **Next** to proceed to the next step.
- The fourth step of the Design Wizard provides a review of the settings you have chosen for the design.




If any of the settings are not as intended, you can click the **Back** button to return to the appropriate step and make any necessary changes.

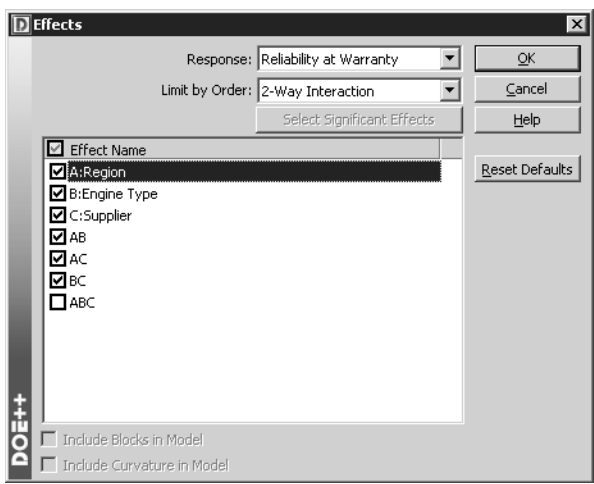
- Click **Finish** to create the new project with the Standard Folio containing the general full factorial design.
- Save the project as “GeneralFullFactorial.rso7.”

- Select **Standard Order** in the **Sort By** area in the Control Panel to sort the Data Sheet by the Standard Order column, then enter the response data in the order given in this example on page 48. The Folio will look like the one shown next.

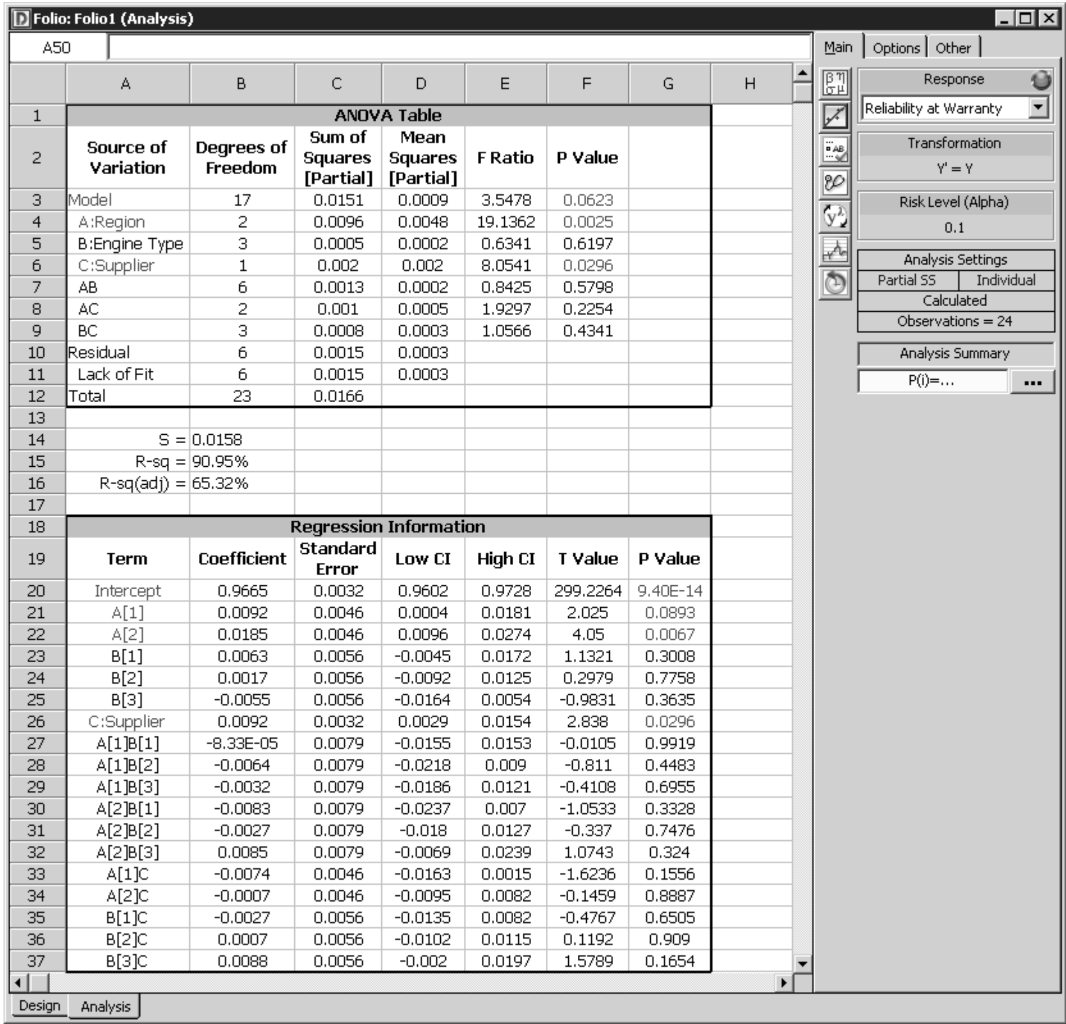



| | Standard Order | Run Order | Block | A:Region | B:Engine Type | C:Supplier | Reliability at Warranty |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 17 | 1 | A | Gasoline | S1 | 0.973 |
| 2 | 2 | 13 | 1 | A | Gasoline | S2 | 0.991 |
| 3 | 3 | 16 | 1 | A | Diesel | S1 | 0.971 |
| 4 | 4 | 5 | 1 | A | Diesel | S2 | 0.971 |
| 5 | 5 | 19 | 1 | A | Hybrid | S1 | 0.978 |
| 6 | 6 | 14 | 1 | A | Hybrid | S2 | 0.956 |
| 7 | 7 | 11 | 1 | A | Ethanol | S1 | 0.988 |
| 8 | 8 | 9 | 1 | A | Ethanol | S2 | 0.978 |
| 9 | 9 | 24 | 1 | B | Gasoline | S1 | 0.988 |
| 10 | 10 | 23 | 1 | B | Gasoline | S2 | 0.978 |
| 11 | 11 | 20 | 1 | B | Diesel | S1 | 0.992 |
| 12 | 12 | 12 | 1 | B | Diesel | S2 | 0.976 |
| 13 | 13 | 21 | 1 | B | Hybrid | S1 | 0.999 |
| 14 | 14 | 22 | 1 | B | Hybrid | S2 | 0.977 |
| 15 | 15 | 7 | 1 | B | Ethanol | S1 | 0.995 |
| 16 | 16 | 1 | 1 | B | Ethanol | S2 | 0.975 |
| 17 | 17 | 4 | 1 | C | Gasoline | S1 | 0.977 |
| 18 | 18 | 15 | 1 | C | Gasoline | S2 | 0.93 |
| 19 | 19 | 2 | 1 | C | Diesel | S1 | 0.971 |
| 20 | 20 | 18 | 1 | C | Diesel | S2 | 0.928 |
| 21 | 21 | 8 | 1 | C | Hybrid | S1 | 0.96 |
| 22 | 22 | 6 | 1 | C | Hybrid | S2 | 0.896 |
| 23 | 23 | 10 | 1 | C | Ethanol | S1 | 0.916 |
| 24 | 24 | 3 | 1 | C | Ethanol | S2 | 0.932 |

- On the Options page of the Control Panel, specify a **Risk Level (Alpha)** value of **0.1** for the Reliability at Warranty response and select to use **Individual Terms**.
- Return to the Main page of the Control Panel and select **Select Effects** from the **Data** menu or click the **Select Effects** icon in the Control Panel.
- In the Effects window, select **2-Way Interaction** from the **Limit by Order** drop-down, as shown next.



- Click **OK** to accept your selections and close the window then click **Calculate**. The Analysis tab is shown next.




| ANOVA Table | | | | | |
|---|---|---|---|---|---|
| Source of Variation | Degrees of Freedom | Sum of Squares [Partial] | Mean Squares [Partial] | F Ratio | P Value |
| Model | 17 | 0.0151 | 0.0009 | 3.5478 | 0.0623 |
| A:Region | 2 | 0.0096 | 0.0048 | 19.1362 | 0.0025 |
| B:Engine Type | 3 | 0.0005 | 0.0002 | 0.6341 | 0.6197 |
| C:Supplier | 1 | 0.002 | 0.002 | 8.0541 | 0.0296 |
| AB | 6 | 0.0013 | 0.0002 | 0.8425 | 0.5798 |
| AC | 2 | 0.001 | 0.0005 | 1.9297 | 0.2254 |
| BC | 3 | 0.0008 | 0.0003 | 1.0566 | 0.4341 |
| Residual | 6 | 0.0015 | 0.0003 | | |
| Lack of Fit | 6 | 0.0015 | 0.0003 | | |
| Total | 23 | 0.0166 | | | |

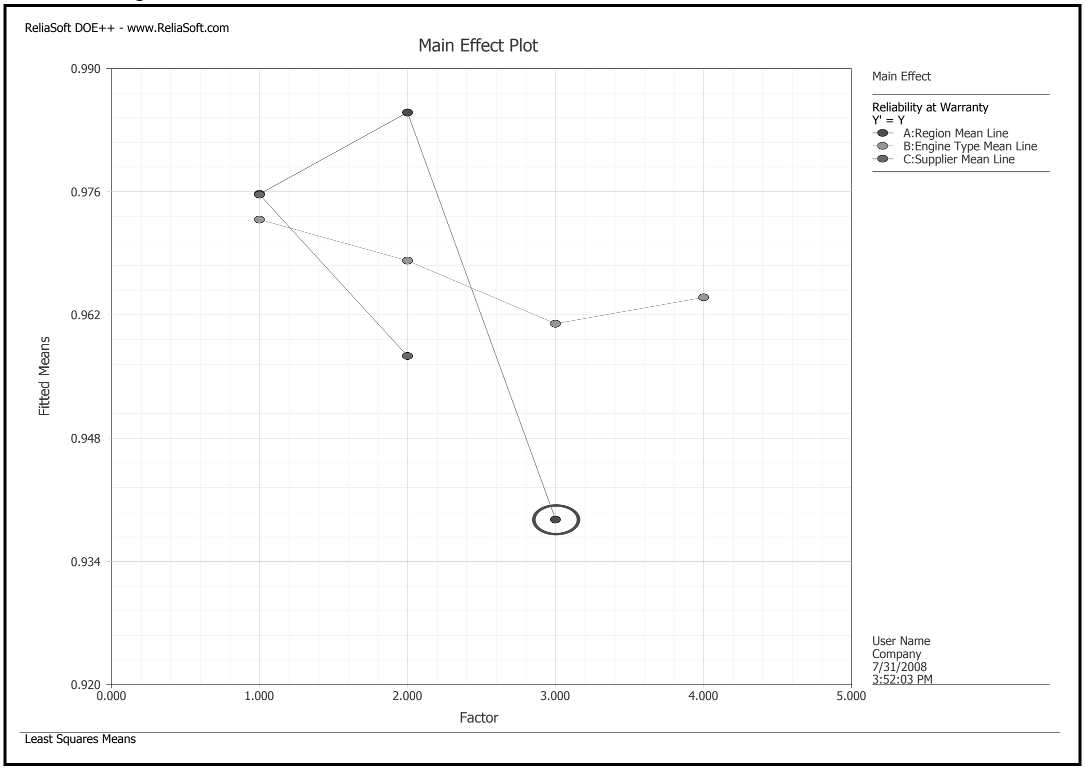
S = 0.0158
R-sq = 90.95%
R-sq(adj) = 65.32%

| Regression Information | | | | | | |
|---|---|---|---|---|---|---|
| Term | Coefficient | Standard Error | Low CI | High CI | T Value | P Value |
| Intercept | 0.9665 | 0.0032 | 0.9602 | 0.9728 | 299.2264 | 9.40E-14 |
| A[1] | 0.0092 | 0.0046 | 0.0004 | 0.0181 | 2.025 | 0.0893 |
| A[2] | 0.0185 | 0.0046 | 0.0096 | 0.0274 | 4.05 | 0.0067 |
| B[1] | 0.0063 | 0.0056 | -0.0045 | 0.0172 | 1.1321 | 0.3008 |
| B[2] | 0.0017 | 0.0056 | -0.0092 | 0.0125 | 0.2979 | 0.7758 |
| B[3] | -0.0055 | 0.0056 | -0.0164 | 0.0054 | -0.9831 | 0.3635 |
| C:Supplier | 0.0092 | 0.0032 | 0.0029 | 0.0154 | 2.838 | 0.0296 |
| A[1]B[1] | -8.33E-05 | 0.0079 | -0.0155 | 0.0153 | -0.0105 | 0.9919 |
| A[1]B[2] | -0.0064 | 0.0079 | -0.0218 | 0.009 | -0.811 | 0.4483 |
| A[1]B[3] | -0.0032 | 0.0079 | -0.0186 | 0.0121 | -0.4108 | 0.6955 |
| A[2]B[1] | -0.0083 | 0.0079 | -0.0237 | 0.007 | -1.0533 | 0.3328 |
| A[2]B[2] | -0.0027 | 0.0079 | -0.018 | 0.0127 | -0.337 | 0.7476 |
| A[2]B[3] | 0.0085 | 0.0079 | -0.0069 | 0.0239 | 1.0743 | 0.324 |
| A[1]C | -0.0074 | 0.0046 | -0.0163 | 0.0015 | -1.6236 | 0.1556 |
| A[2]C | -0.0007 | 0.0046 | -0.0095 | 0.0082 | -0.1459 | 0.8887 |
| B[1]C | -0.0027 | 0.0056 | -0.0135 | 0.0082 | -0.4767 | 0.6505 |
| B[2]C | 0.0007 | 0.0056 | -0.0102 | 0.0115 | 0.1192 | 0.909 |
| B[3]C | 0.0088 | 0.0056 | -0.002 | 0.0197 | 1.5789 | 0.1654 |

From the ANOVA table, the region and the supplier are identified as significant factors at the 10% significance level (indicated by red text in the Source of Variation and P Value columns). The engine type is not found to affect the reliability of the units.

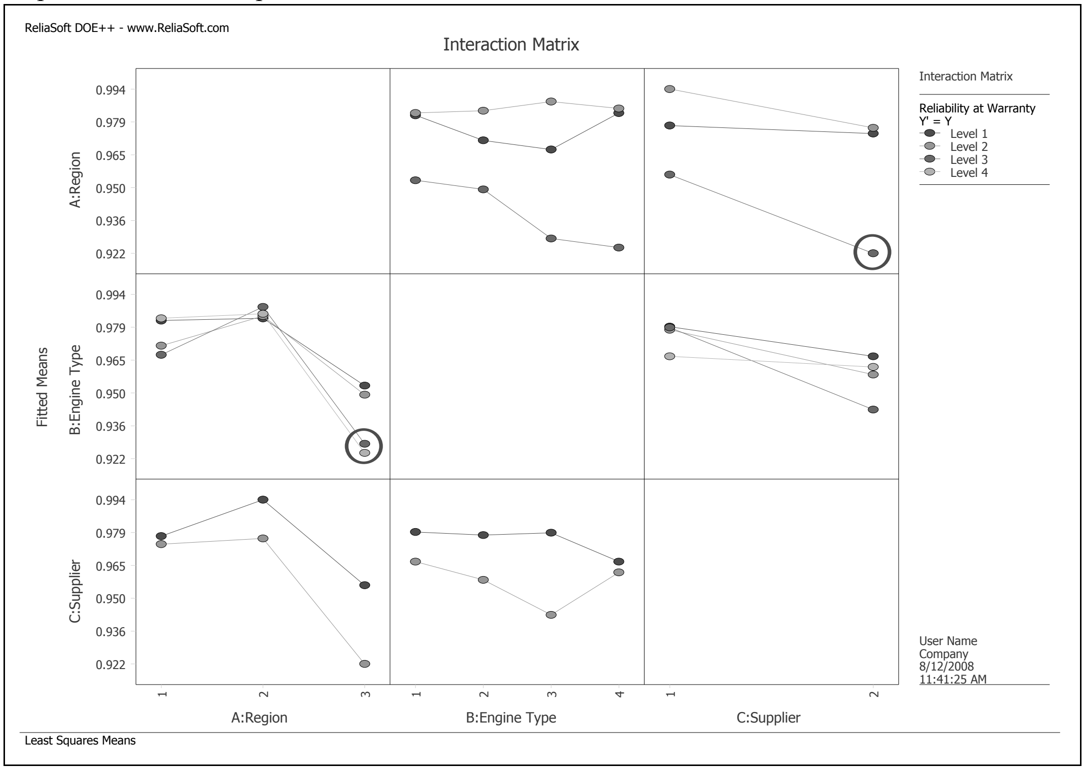
- Click the **Plot** icon to add the Plot tab to the Folio.

- Select **Main Effects** from the **Plot Type** menu in the Control Panel. The Main Effects plot is shown next. Note that the circle around one point in this plot is an annotation created in ReliaSoft Draw to emphasize the relevant point.




This plot shows the individual factors versus reliability at warranty. The critical population is the one with the lowest reliability. You can see that the population deployed in region C had the lowest reliability. To refine this conclusion, you can view the Interaction Matrix plot.

- Select **Interaction Matrix** from the **Plot Type** menu in the Control Panel. The Interaction Matrix plot is shown next. Note that the circles around points in this plot are annotations created in ReliaSoft Draw to emphasize the relevant points.




The plot at the upper right shows the effect of the interaction between region and supplier on reliability. To determine what population is represented by each point, hover the cursor over the point. You can see that the circled point represents the population deployed in the third region (region C) using parts from the second supplier (supplier S2). This population had particularly low reliability. By examining the plot at the middle left, you can see that the population deployed in region C with hybrid and ethanol engines also had very low reliability.

- Save and close the project then proceed to the next example.

## 4.5 Example 5: Taguchi Orthogonal Array Design

In plastic processing, an engineer would like to find the effect of different factors on the impact strength of the material. Four process variables are to be studied: the holding pressure, the injection speed, the barrel temperature and the mold temperature. The engineer would like to explore three settings for each factor, so a general full factorial design would require $3^4 = 81$ runs to perform. Given time and resource constraints, it is decided to carry out a Taguchi L9 design instead, where higher interaction terms will be assumed to be negligible.

The following parameter settings are used:[6]

| Parameter | Units | Level 1 | Level 2 | Level 3 |
|---|---|---|---|---|
| Hold pressure | bar | 250 | 300 | 350 |
| Injection speed | mm/s | 75 | 100 | 125 |
| Barrel temperature | °C | 200 | 230 | 160 |
| Mold temperature | °C | 30 | 40 | 50 |

The following impact strength data are collected:

| Hold Pressure (bar) | Injection Speed (mm/x) | Barrel Temperature (°C) | Mold Temperature (°C) | Impact Strength (kJ/m$^2$) |
|---|---|---|---|---|
| 250 | 75 | 200 | 30 | 87.48 |
| 250 | 100 | 230 | 40 | 86.54 |
| 250 | 125 | 160 | 50 | 85.72 |
| 300 | 75 | 230 | 50 | 87.93 |
| 300 | 100 | 160 | 30 | 86.48 |
| 300 | 125 | 200 | 40 | 85.31 |
| 350 | 75 | 160 | 40 | 87.89 |
| 350 | 100 | 200 | 50 | 86.11 |
| 350 | 125 | 230 | 30 | 85.75 |

Do the following:

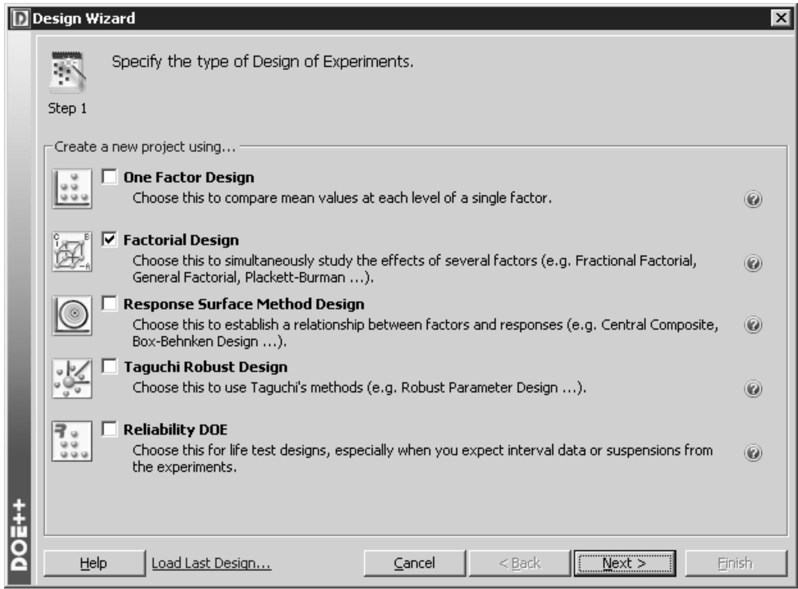
- Identify the significant factors.
- Identify the factor settings that will maximize the impact strength.

*The sample file for this example is located in the "Training Guide" folder in your application directory (e.g. C:\Program Files\ReliaSoft\DOE\Training Guide) and is named "TaguchiOA.rdoe. The data set for this example is available in the Spreadsheet in the "TaguchiOA-Data.rso7" file located in the "Raw Data" folder within the "Training Guide" folder."*

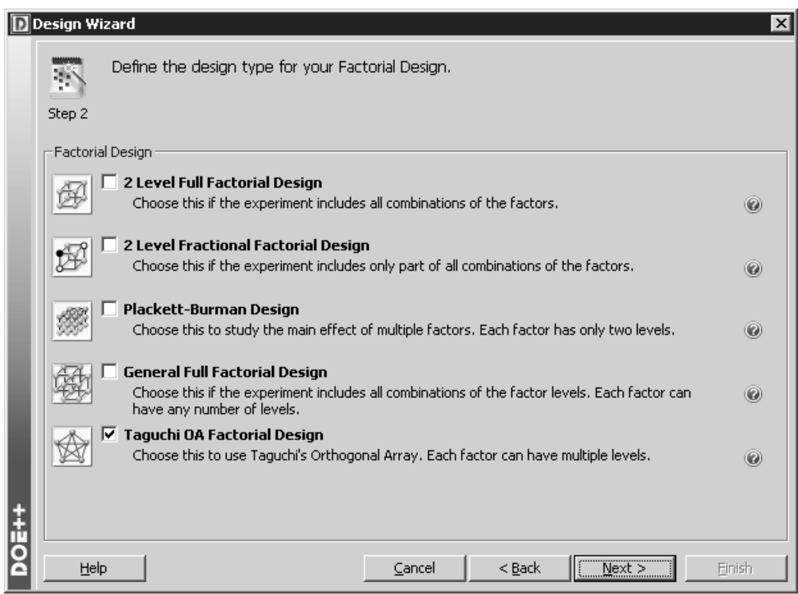
6. http://www.tekes.fi/julkaisut/propros/raportti57.pdf

**Solution**

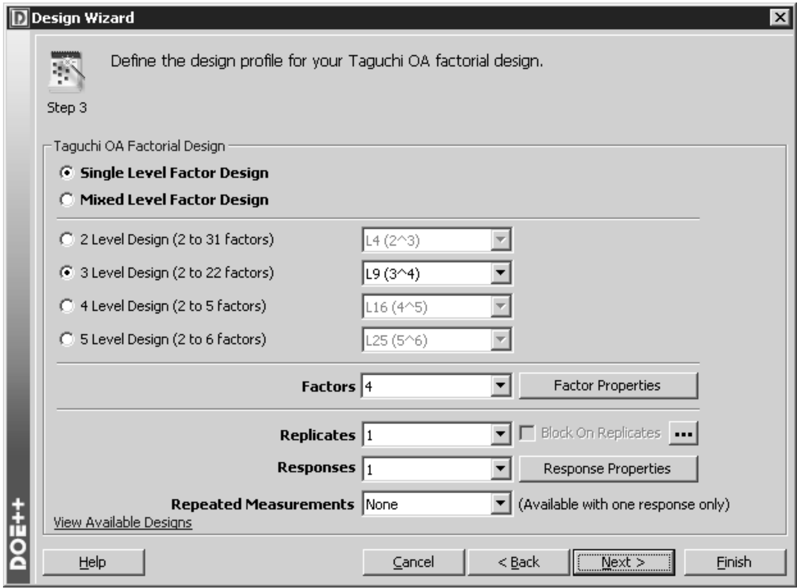
- Create a new project. On the first page of the Design Wizard, select to create a **Factorial Design** and click **Next** to proceed to the next step.



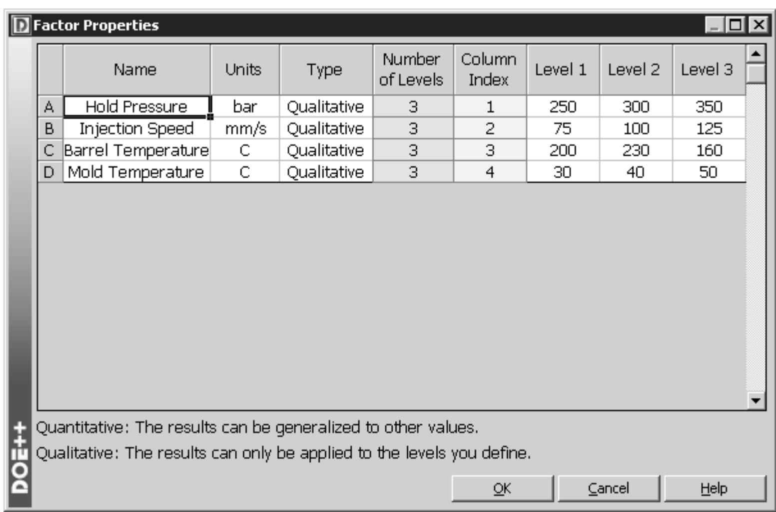

- In the second step of the Design Wizard, select **Taguchi OA Factorial Design** and click **Next** to proceed to the next step.



- In the third step of the Design Wizard, select to create a **Single Level Factor Design**. Select **3 Level Design** and then select **L9(3^4)** from the drop-down and leave all other fields at their default settings, as shown next.

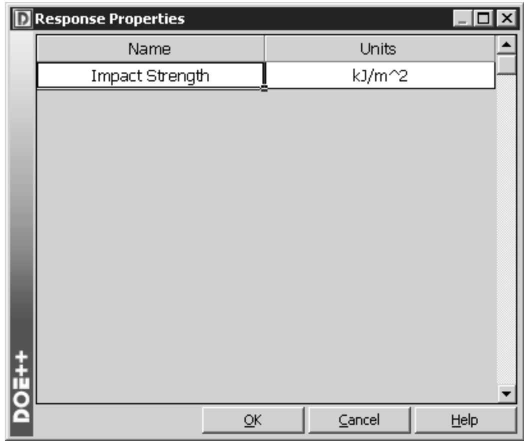



- Click the **Factor Properties** button. In the Factor Properties window that appears, enter the names and units for the factors then specify the number of levels for each factor and enter the level values, as shown next.

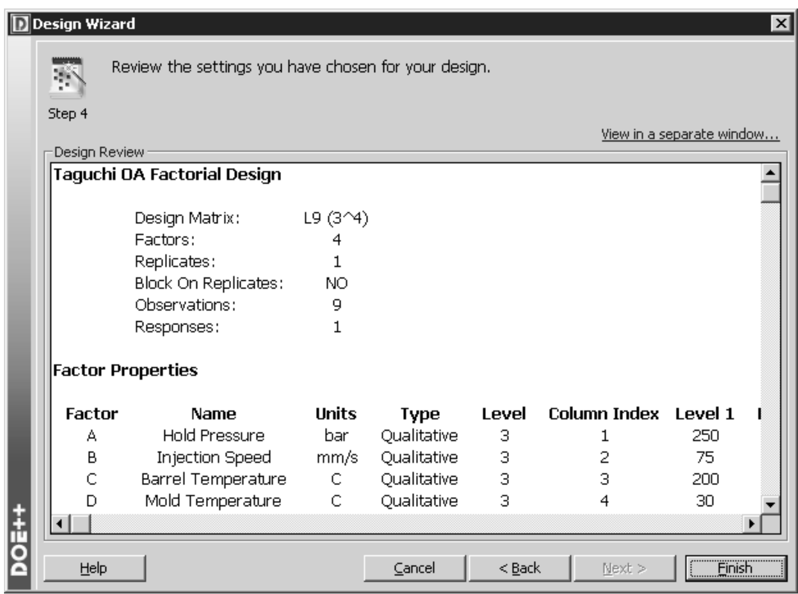



- Click **OK** to close the Factor Properties window and return to the Design Wizard.

- Click the **Response Properties** button. In the Response Properties window that appears, define the response name, as shown next.



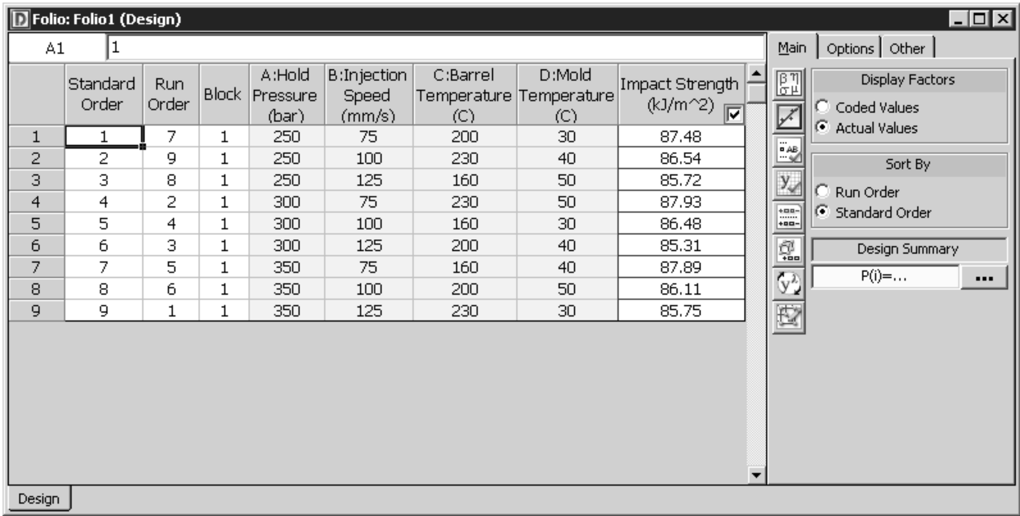

- Click **OK** to close the Response Properties window and return to the Design Wizard.
- Click **Next** to proceed to the next step.
- The fourth step of the Design Wizard provides a review of the settings you have chosen for the design.




If any of the settings are not as intended, you can click the **Back** button to return to the appropriate step and make any necessary changes.

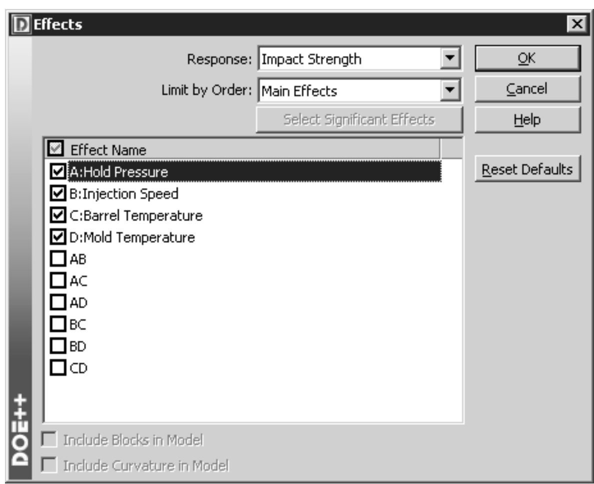
- Click **Finish** to create the new project with the Standard Folio containing the Taguchi OA design.
- Save the project as “TaguchiOA.rso7.”

- Select **Standard Order** in the **Sort By** area in the Control Panel to sort the Data Sheet by the Standard Order column, then enter the response data in the order given in this example on page 56. The Folio will look like the one shown next.

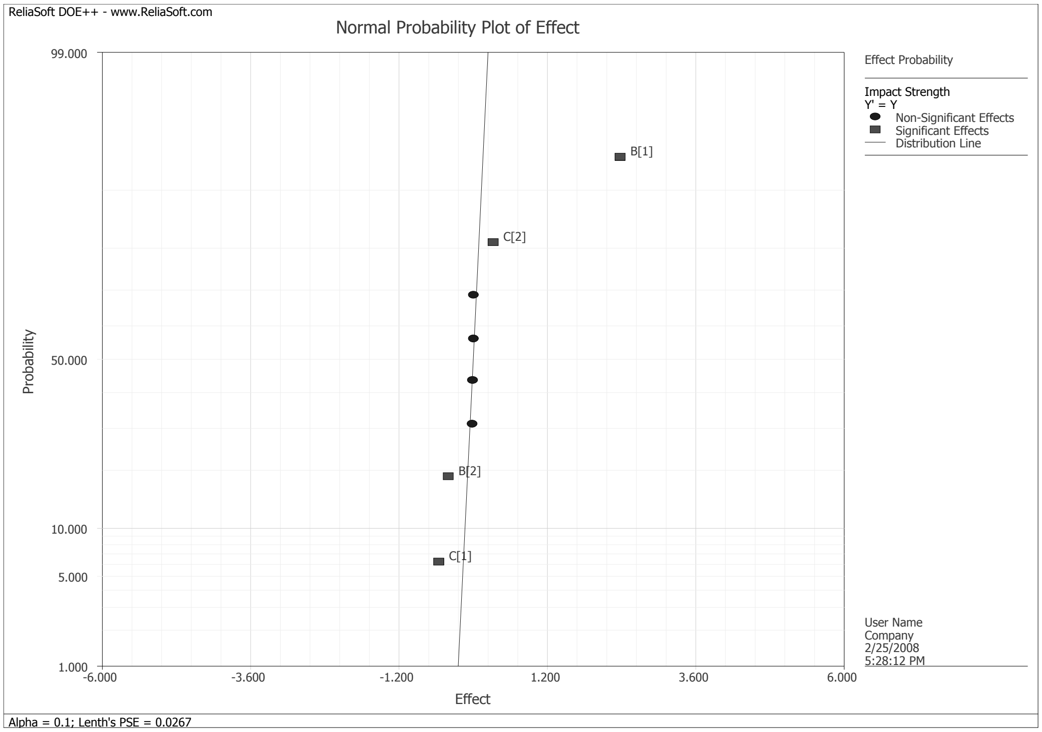



| | Standard Order | Run Order | Block | A:Hold Pressure (bar) | B:Injection Speed (mm/s) | C:Barrel Temperature (C) | D:Mold Temperature (C) | Impact Strength (kJ/m^2) |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 7 | 1 | 250 | 75 | 200 | 30 | 87.48 |
| 2 | 2 | 9 | 1 | 250 | 100 | 230 | 40 | 86.54 |
| 3 | 3 | 8 | 1 | 250 | 125 | 160 | 50 | 85.72 |
| 4 | 4 | 2 | 1 | 300 | 75 | 230 | 50 | 87.93 |
| 5 | 5 | 4 | 1 | 300 | 100 | 160 | 30 | 86.48 |
| 6 | 6 | 3 | 1 | 300 | 125 | 200 | 40 | 85.31 |
| 7 | 7 | 5 | 1 | 350 | 75 | 160 | 40 | 87.89 |
| 8 | 8 | 6 | 1 | 350 | 100 | 200 | 50 | 86.11 |
| 9 | 9 | 1 | 1 | 350 | 125 | 230 | 30 | 85.75 |

- On the Options page of the Control Panel, specify a **Risk Level (Alpha)** value of **0.1** for the Impact Strength response and select to use **Individual Terms**.
- Return to the Main page of the Control Panel and select **Select Effects** from the **Data** menu or click the **Select Effects** icon in the Control Panel.
- In the Effects window, select **Main Effects** from the **Limit by Order** drop-down, as shown next.

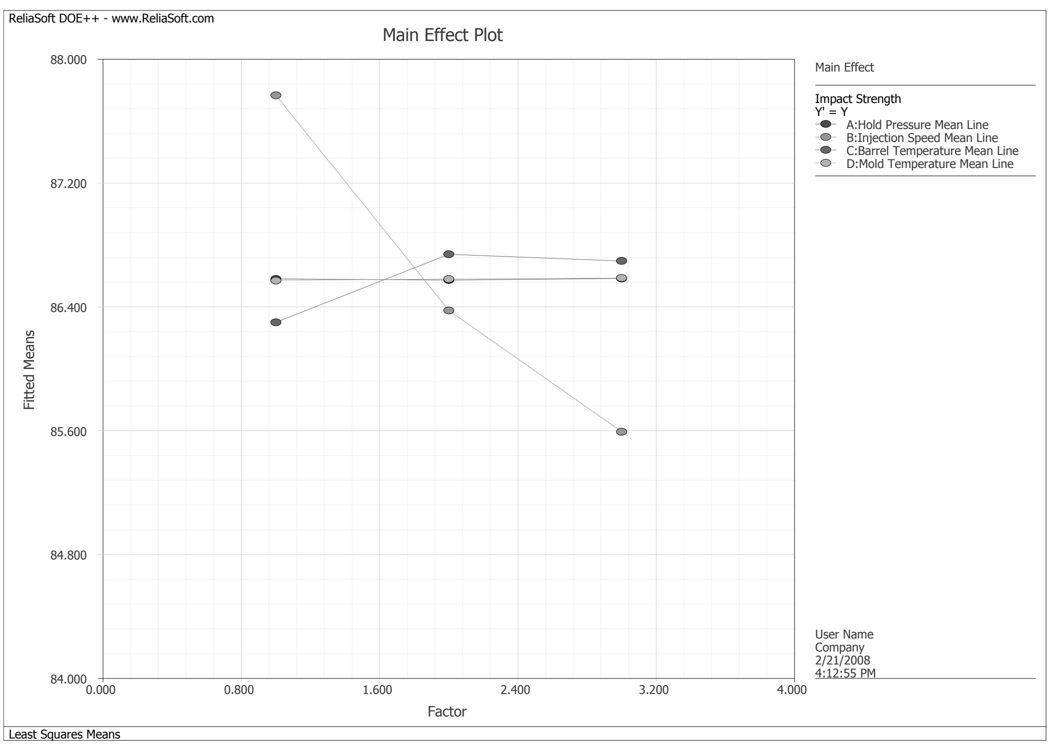


- Click **OK** to accept your selections and close the window then click **Plot**. The Effect Probability plot is shown next.




Factors B and C are found to affect the response at a significance level of 0.1. Note that no replicates exist, so these factors are identified using Lenth's PSE of 0.0267.

- Select **Main Effects** from the **Plot Type** menu in the Control Panel. The Main Effects plot is shown next.



From this plot, you can see that factor B should be set at level 1 (injection speed = 75 mm/s) and factor C should be set at level 2 (barrel temperature = 230°C). Factors A and D, the hold pressure and the mold temperature, should be set at the most economical/feasible conditions, as they do not affect the process.

- Save and close the project then proceed to the next example.

## 4.6 Example 6: Response Surface Methodology: Box-Behnken Design

In order to measure the amount of a contaminant in certain foods, a standard monitoring system exists. A chemist would like to propose an alternate method of quality control in which the contaminant reacts with reactants A and B in the presence of catalyst C. The product of this reaction gives a simpler, cheaper method to quantify the presence of the contaminant. The experimenter would like to find the optimum amount of reactants A and B and the catalyst, such that the detection error is minimized. The detection error is measured in terms of percent deviation from the results obtained from the currently used procedure. In this experiment, the experimental solution containing the contaminant will be kept at the maximum adequate dose. The levels of the reactants and catalyst are shown next.

| Factor | Low Level | High Level |
|---|---|---|
| Reactant A (M) | 1.00E-05 | 1.00E-04 |
| Reactant B (M) | 1.00E-06 | 1.00E-05 |
| Catalyst (M) | 1.00E-03 | 5.00E-03 |

The experimenter chooses to run a Box-Behnken design after having determined during prior investigation that there is a quadratic relation between the different factors and the response.

The data set is shown next.

| Reactant A (M) | Reactant B (M) | Catalyst (M) | Percent Error (%) |
|---|---|---|---|
| -1 | -1 | 0 | 1.596 |
| 1 | -1 | 0 | 1.36 |
| -1 | 1 | 0 | 1.702 |
| 1 | 1 | 0 | 1.066 |
| -1 | 0 | -1 | 1.904 |
| 1 | 0 | -1 | 1.526 |
| -1 | 0 | 1 | 1.498 |
| 1 | 0 | 1 | 0.102 |
| 0 | -1 | -1 | 1.25 |
| 0 | 1 | -1 | 1.516 |
| 0 | -1 | 1 | 1.278 |
| 0 | 1 | 1 | 0.886 |

| Reactant A (M) | Reactant B (M) | Catalyst (M) | Percent Error (%) |
|---|---|---|---|
| 0 | 0 | 0 | 0.728 |
| 0 | 0 | 0 | 0.538 |
| 0 | 0 | 0 | 1.038 |

Note that for ease of viewing the factor values in this data set are expressed as coded values, where -1 represents the factor's low value and 1 represents the factor's high value.

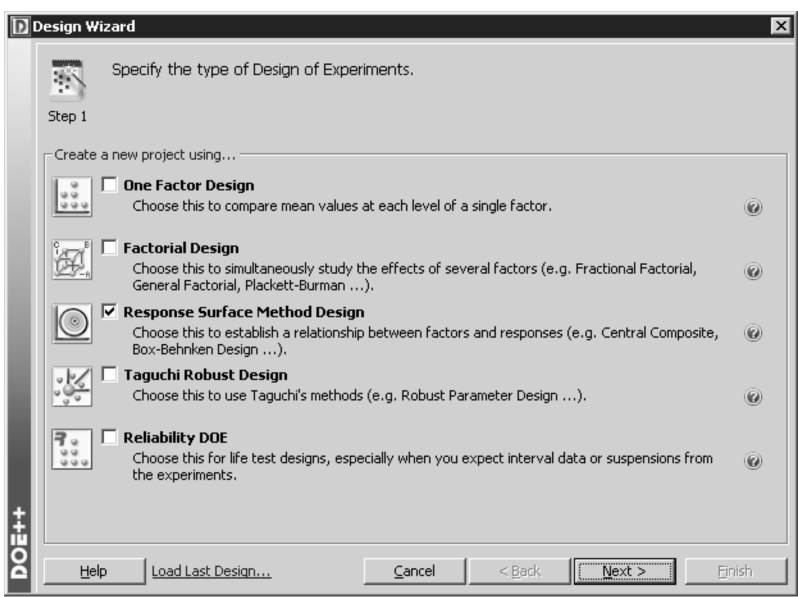
Do the following:

- Find an appropriate model for the response.
- Find settings for the factors such that the percent error is minimized (0.3% is desired but up to 0.5% is acceptable).

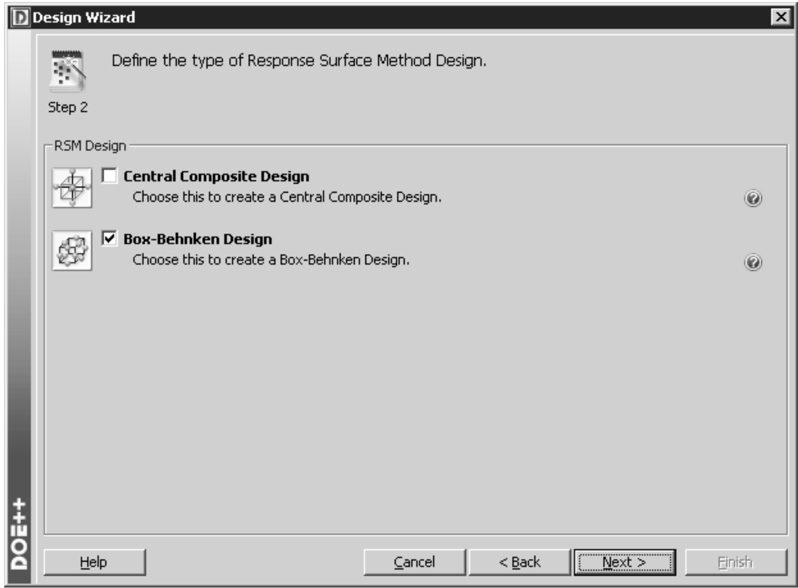
*The sample file for this example is located in the "Training Guide" folder in your application directory (e.g. C:\Program Files\ReliaSoft\DOE\Training Guide) and is named "BoxBehnken.rdoe." The data set for this example is available in the Spreadsheet in the "BoxBehnken-Data.rso7" file located in the "Raw Data" folder within the "Training Guide" folder.*

**Solution**

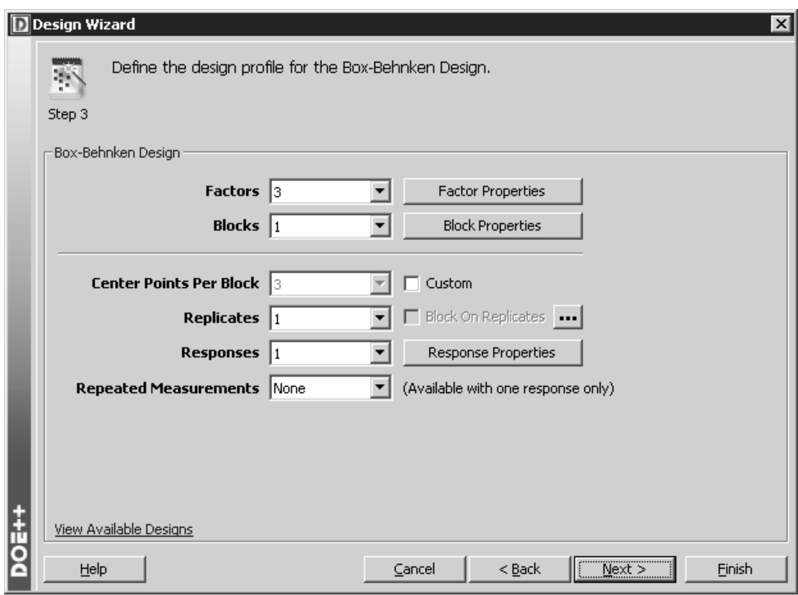
- Create a new project. On the first page of the Design Wizard, select to create a **Response Surface Method Design** and click **Next** to proceed to the next step.



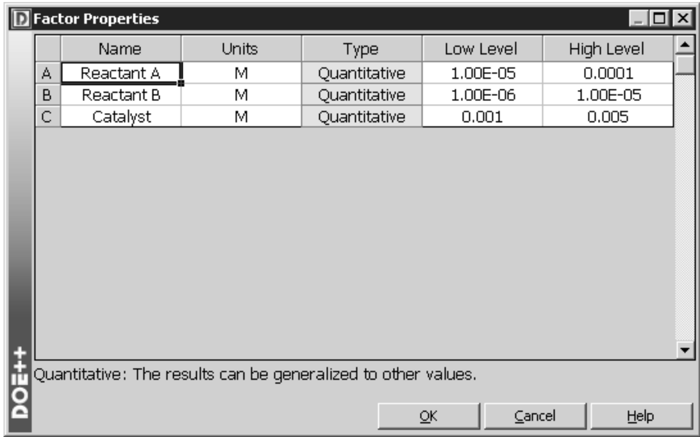
- In the second step of the Design Wizard, select **Box-Behnken Design** and click **Next** to proceed to the next step.




- In the third step of the Design Wizard, select **3** from the Factors drop-down and leave all other fields at their default settings, as shown next.

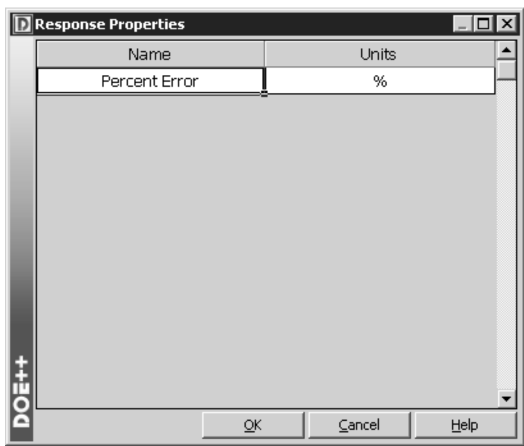


- Click the **Factor Properties** button. In the Factor Properties window that appears, enter the names, units and high and low level values for the factors, as shown next.



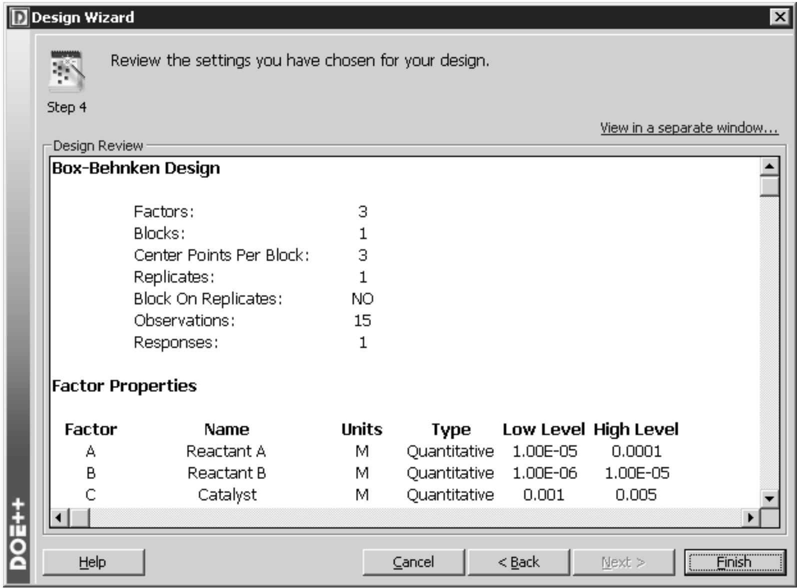

Note that all factors in Box-Behnken designs are quantitative.

- Click **OK** to close the Factor Properties window and return to the Design Wizard.
- Click the **Response Properties** button. In the Response Properties window that appears, define the response name, as shown next.



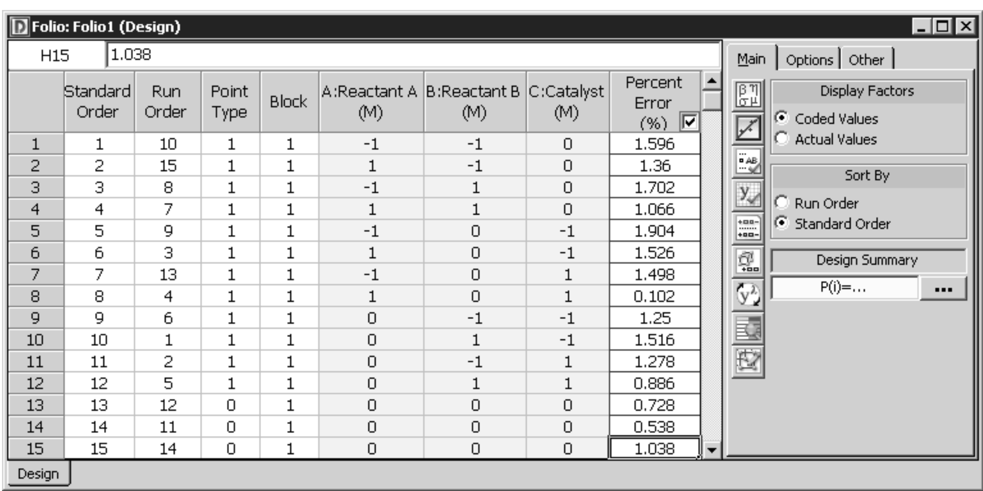

- Click **OK** to close the Response Properties window and return to the Design Wizard.
- Click **Next** to proceed to the next step.

- The fourth step of the Design Wizard provides a review of the settings you have chosen for the design.




If any of the settings are not as intended, you can click the **Back** button to return to the appropriate step and make any necessary changes.

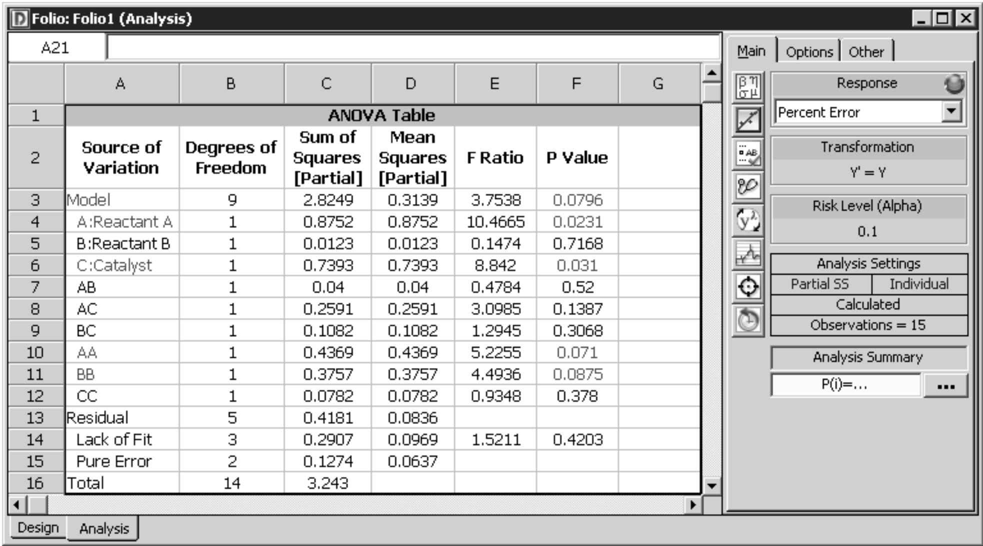
- Click **Finish** to create the new project with the Standard Folio containing the Box-Behnken design.
- Save the project as “BoxBehnken.rso7.”
- Select **Coded Values** in the **Display Factors** area in the Control Panel to view the factor values in their coded form.
- Select **Standard Order** in the **Sort By** area in the Control Panel to sort the Data Sheet by the Standard Order column, then enter the response data in the order given in this example on page 62. The Folio will look like the one shown next.




| | Standard Order | Run Order | Point Type | Block | A:Reactant A (M) | B:Reactant B (M) | C:Catalyst (M) | Percent Error (%) |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 10 | 1 | 1 | -1 | -1 | 0 | 1.596 |
| 2 | 2 | 15 | 1 | 1 | 1 | -1 | 0 | 1.36 |
| 3 | 3 | 8 | 1 | 1 | -1 | 1 | 0 | 1.702 |
| 4 | 4 | 7 | 1 | 1 | 1 | 1 | 0 | 1.066 |
| 5 | 5 | 9 | 1 | 1 | -1 | 0 | -1 | 1.904 |
| 6 | 6 | 3 | 1 | 1 | 1 | 0 | -1 | 1.526 |
| 7 | 7 | 13 | 1 | 1 | -1 | 0 | 1 | 1.498 |
| 8 | 8 | 4 | 1 | 1 | 1 | 0 | 1 | 0.102 |
| 9 | 9 | 6 | 1 | 1 | 0 | -1 | -1 | 1.25 |
| 10 | 10 | 1 | 1 | 1 | 0 | 1 | -1 | 1.516 |
| 11 | 11 | 2 | 1 | 1 | 0 | -1 | 1 | 1.278 |
| 12 | 12 | 5 | 1 | 1 | 0 | 1 | 1 | 0.886 |
| 13 | 13 | 12 | 0 | 1 | 0 | 0 | 0 | 0.728 |
| 14 | 14 | 11 | 0 | 1 | 0 | 0 | 0 | 0.538 |
| 15 | 15 | 14 | 0 | 1 | 0 | 0 | 0 | 1.038 |

- On the Options page of the Control Panel, specify a **Risk Level (Alpha)** value of **0.1** for the Percent Error response and select to use **Individual Terms**.

- Return to the Main page of the Control Panel and click **Calculate**. The ANOVA table on the Analysis tab is shown next.

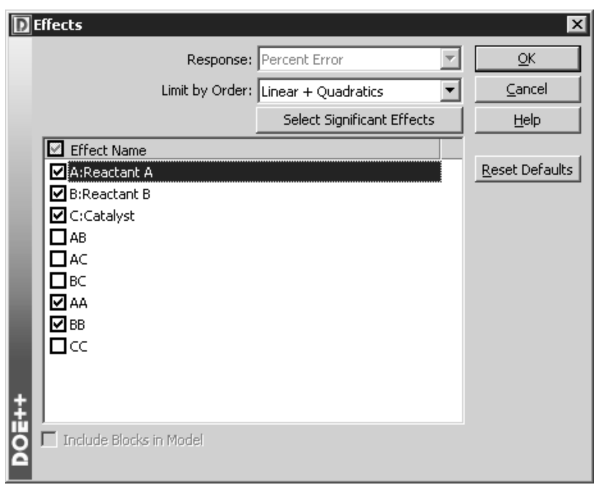


| Source of Variation | Degrees of Freedom | Sum of Squares [Partial] | Mean Squares [Partial] | F Ratio | P Value |
|---|---|---|---|---|---|
| Model | 9 | 2.8249 | 0.3139 | 3.7538 | 0.0796 |
| A:Reactant A | 1 | 0.8752 | 0.8752 | 10.4665 | 0.0231 |
| B:Reactant B | 1 | 0.0123 | 0.0123 | 0.1474 | 0.7168 |
| C:Catalyst | 1 | 0.7393 | 0.7393 | 8.842 | 0.031 |
| AB | 1 | 0.04 | 0.04 | 0.4784 | 0.52 |
| AC | 1 | 0.2591 | 0.2591 | 3.0985 | 0.1387 |
| BC | 1 | 0.1082 | 0.1082 | 1.2945 | 0.3068 |
| AA | 1 | 0.4369 | 0.4369 | 5.2255 | 0.071 |
| BB | 1 | 0.3757 | 0.3757 | 4.4936 | 0.0875 |
| CC | 1 | 0.0782 | 0.0782 | 0.9348 | 0.378 |
| Residual | 5 | 0.4181 | 0.0836 | | |
| Lack of Fit | 3 | 0.2907 | 0.0969 | 1.5211 | 0.4203 |
| Pure Error | 2 | 0.1274 | 0.0637 | | |
| Total | 14 | 3.243 | | | |

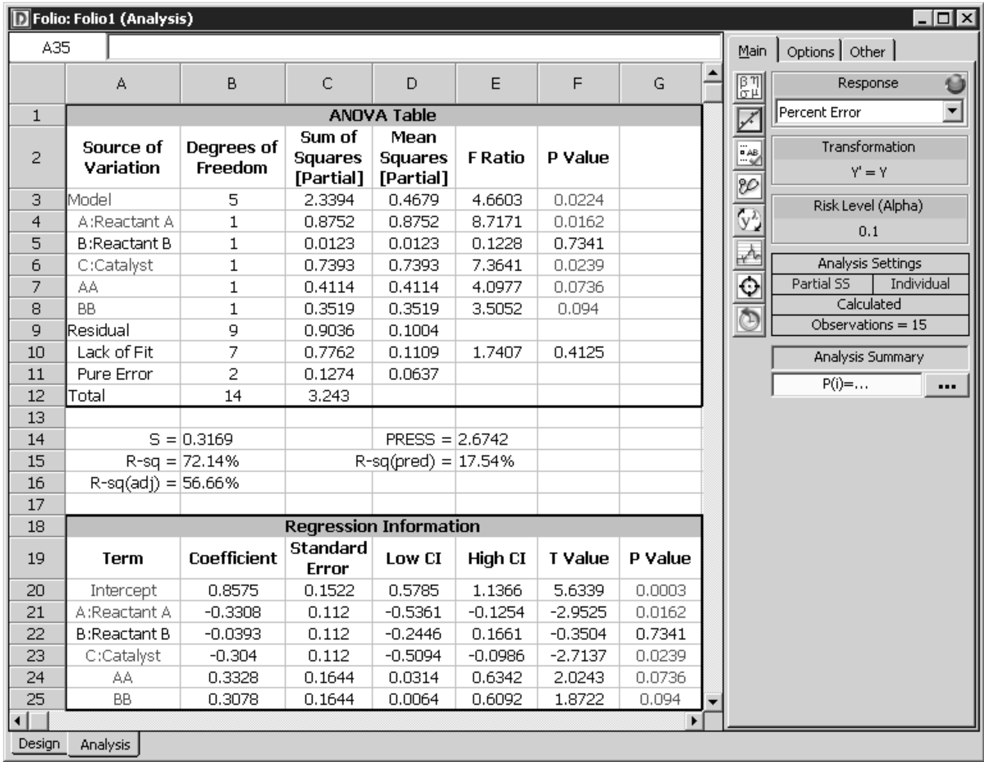


From the ANOVA table, the significant effects are A, C, AA and BB.

- Select **Select Effects** from the **Data** menu or click the **Select Effects** icon in the Control Panel.
- In the Effects window, click the **Select Significant Effects** button to select only the effects shown to be significant. Note that in order for **BB** to be selected, **B** must also be selected so that model is hierarchical, as shown next.



- Click **OK** to accept your selections and close the window then click the **Calculate** icon to re-analyze the data set using the selections you have just made. The changed Analysis tab is shown next.

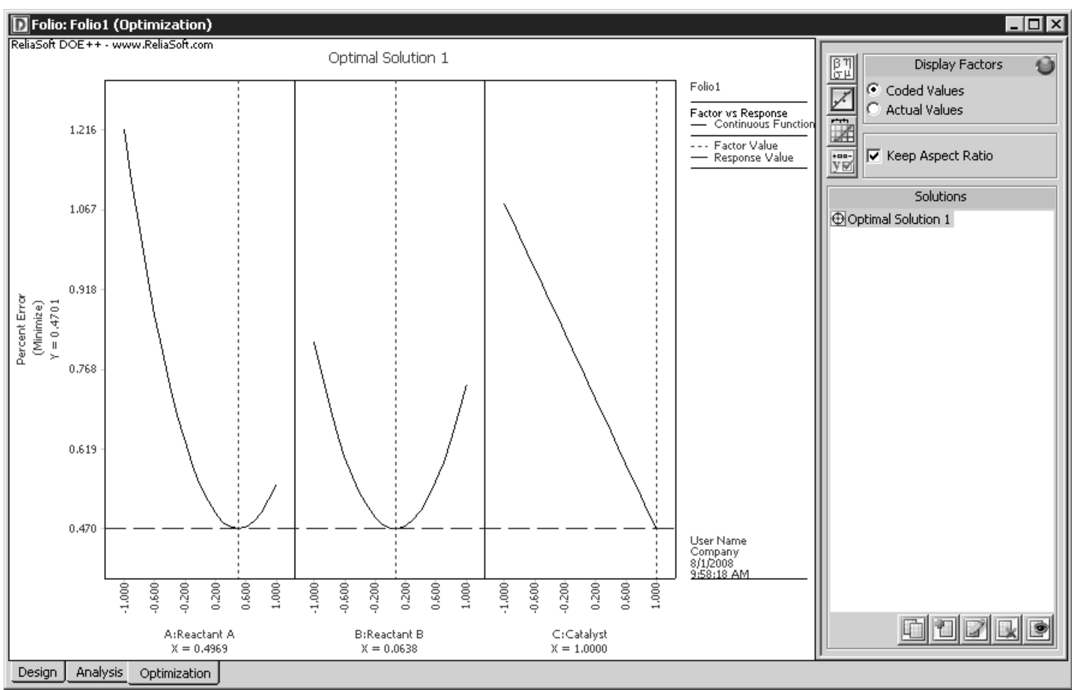



| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | ANOVA Table | | | | | | |
| 2 | Source of Variation | Degrees of Freedom | Sum of Squares [Partial] | Mean Squares [Partial] | F Ratio | P Value | |
| 3 | Model | 5 | 2.3394 | 0.4679 | 4.6603 | 0.0224 | |
| 4 | A:Reactant A | 1 | 0.8752 | 0.8752 | 8.7171 | 0.0162 | |
| 5 | B:Reactant B | 1 | 0.0123 | 0.0123 | 0.1228 | 0.7341 | |
| 6 | C:Catalyst | 1 | 0.7393 | 0.7393 | 7.3641 | 0.0239 | |
| 7 | AA | 1 | 0.4114 | 0.4114 | 4.0977 | 0.0736 | |
| 8 | BB | 1 | 0.3519 | 0.3519 | 3.5052 | 0.094 | |
| 9 | Residual | 9 | 0.9036 | 0.1004 | | | |
| 10 | Lack of Fit | 7 | 0.7762 | 0.1109 | 1.7407 | 0.4125 | |
| 11 | Pure Error | 2 | 0.1274 | 0.0637 | | | |
| 12 | Total | 14 | 3.243 | | | | |
| 13 | | | | | | | |
| 14 | S = | 0.3169 | | PRESS = | 2.6742 | | |
| 15 | R-sq = | 72.14% | | R-sq(pred) = | 17.54% | | |
| 16 | R-sq(adj) = | 56.66% | | | | | |
| 17 | | | | | | | |
| 18 | Regression Information | | | | | | |
| 19 | Term | Coefficient | Standard Error | Low CI | High CI | T Value | P Value |
| 20 | Intercept | 0.8575 | 0.1522 | 0.5785 | 1.1366 | 5.6339 | 0.0003 |
| 21 | A:Reactant A | -0.3308 | 0.112 | -0.5361 | -0.1254 | -2.9525 | 0.0162 |
| 22 | B:Reactant B | -0.0393 | 0.112 | -0.2446 | 0.1661 | -0.3504 | 0.7341 |
| 23 | C:Catalyst | -0.304 | 0.112 | -0.5094 | -0.0986 | -2.7137 | 0.0239 |
| 24 | AA | 0.3328 | 0.1644 | 0.0314 | 0.6342 | 2.0243 | 0.0736 |
| 25 | BB | 0.3078 | 0.1644 | 0.0064 | 0.6092 | 1.8722 | 0.094 |

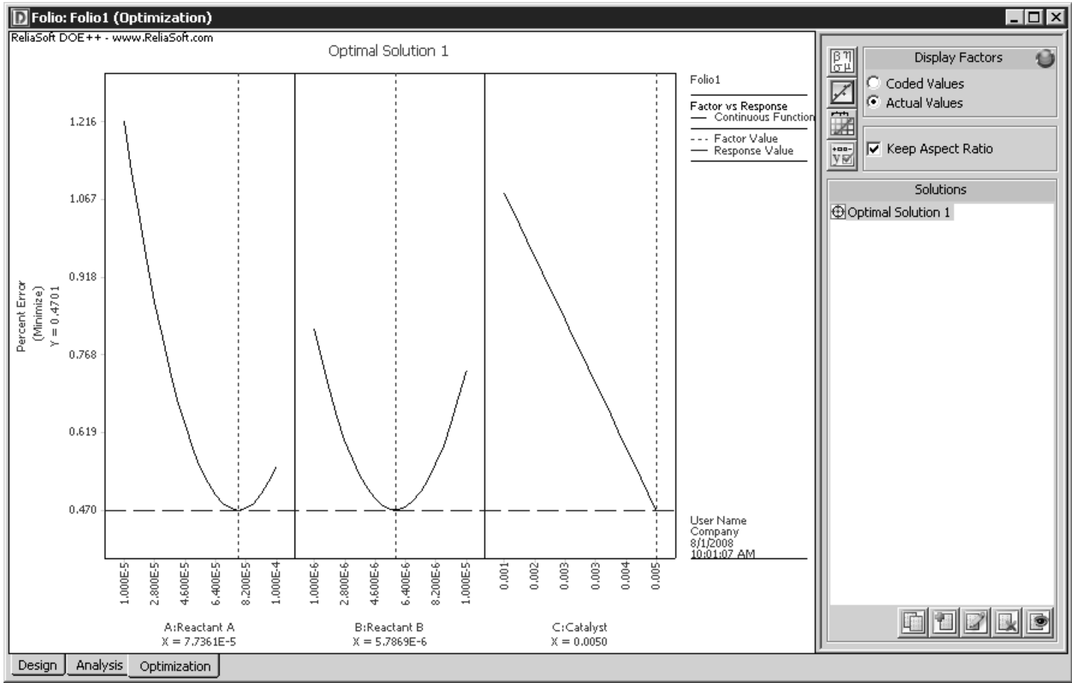
- Select **Optimization** from the **Data** menu or click the **Optimization** icon in the Control Panel.

- In the Optimization Settings window that appears, select **Minimize** from the drop-down in the Goal column and enter **0.3** for the Target value and **0.5** for the Upper Limit value. The Optimization Settings window will look like the one shown next.


- Click **OK** to accept your settings and create the Optimal Solution plot, as shown next.



Note that the factor settings shown are coded values. To view the actual factor values, select **Actual Values** in the Display Factors area in the Control Panel. The solution will be replotted, as shown next.




The optimum settings are found to be reactant A = 7.7361E-5, reactant B = 5.7869E-6 and catalyst C = 0.0050.

- Save and close the project then proceed to the next example.

## 4.7 Example 7: Sequential Optimization

This multi-part example guides you through the process of moving from the present operating conditions to the vicinity of the operating conditions where the response is optimum, then analyzing these ranges using response surface methodology and optimizing the factor settings for the desired response.

The yield from a chemical process is found to be affected by two factors: reaction temperature and reaction time. The current reaction temperature is 230$^o$F and the reaction time is 65 minutes. The experimenter wants to determine the settings of the two factors such that maximum yield can be obtained from the process.

*The sample file for this example is located in the "Training Guide" folder in your application directory (e.g. C:\Program Files\ReliaSoft\DOE\Training Guide) and is named "SequentialOptimization.rdoe." The data sets for this example are available on separate sheets in the Spreadsheet in the "SequentialOptimization-Data.rso7" file located in the "Raw Data" folder within the "Training Guide" folder.*

### 4.7.1 Part 1: Current Operating Conditions

To explore the region around the current operating conditions, the experimenter decides to use a single replicate of the $2^2$ design. Ranges for the factors for this design are chosen to be (225, 235)$^o$F for the reaction temperature and (55, 75) minutes for the reaction time. Five replicates at the center point are also added to the design to estimate error sum of squares and check for model adequacy. The response values obtained for this design are shown below:

| Center Point | Temperature (F) | Reaction Time (min) | Yield (%) |
|---|---|---|---|
| No | 225 | 55 | 33.95 |
| No | 235 | 55 | 36.35 |
| No | 225 | 75 | 35 |
| No | 235 | 75 | 37.25 |
| Yes | 230 | 65 | 35.45 |
| Yes | 230 | 65 | 35.75 |
| Yes | 230 | 65 | 36.05 |
| Yes | 230 | 65 | 35.3 |
| Yes | 230 | 65 | 35.9 |

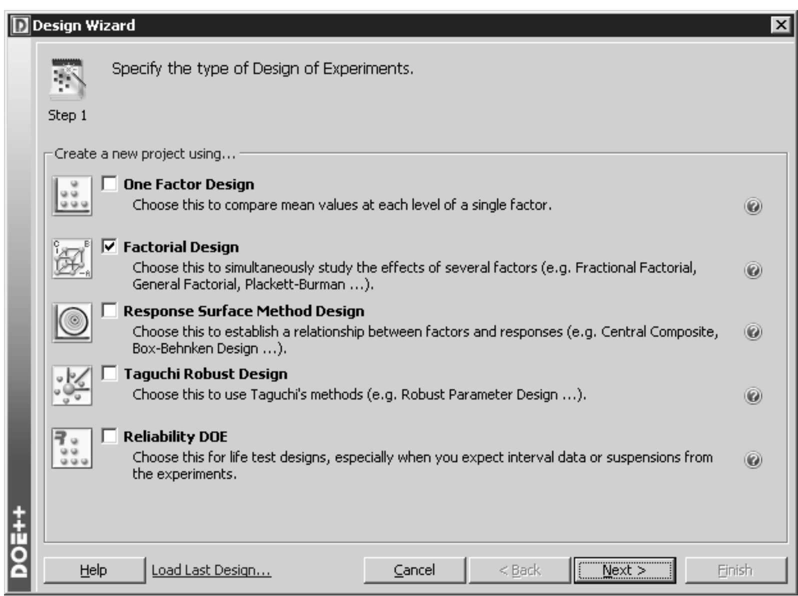
Do the following:

- Find an appropriate model for this response.

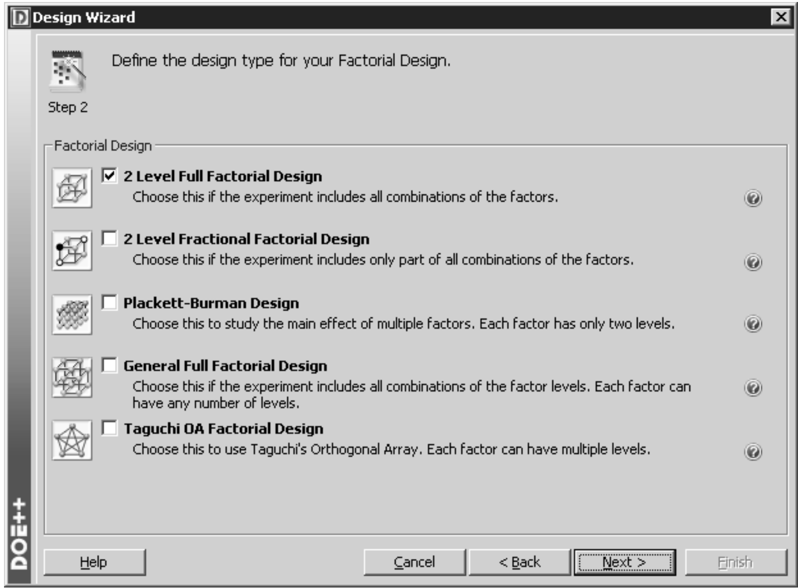
*In the "Sequential Optimization.rdoe" sample file, the Folio for this part of the example is named "Folio1."*

**Solution**

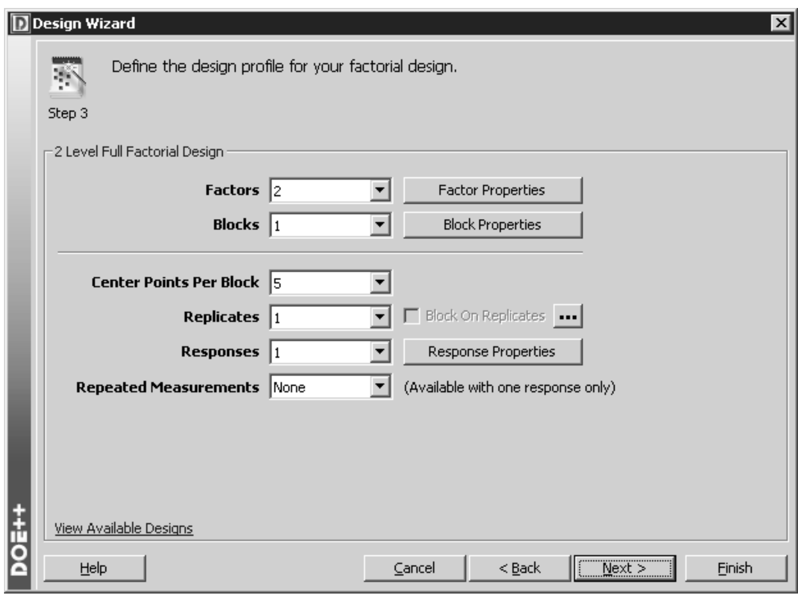
- Create a new project. On the first page of the Design Wizard, select to create a **Factorial Design** and click **Next** to proceed to the next step.



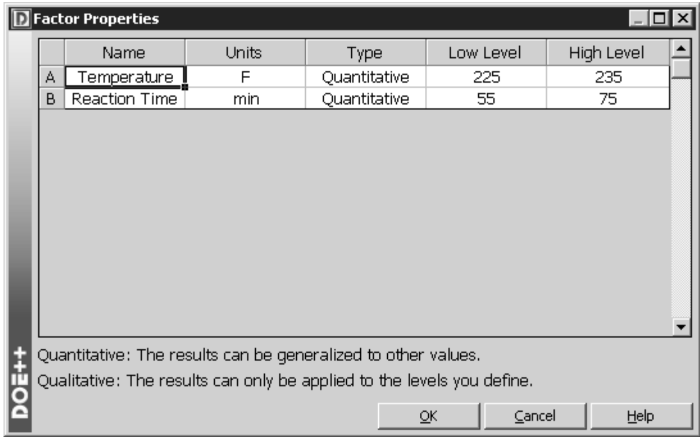
- In the second step of the Design Wizard, select **2 Level Full Factorial Design** and click **Next** to proceed to the next step.



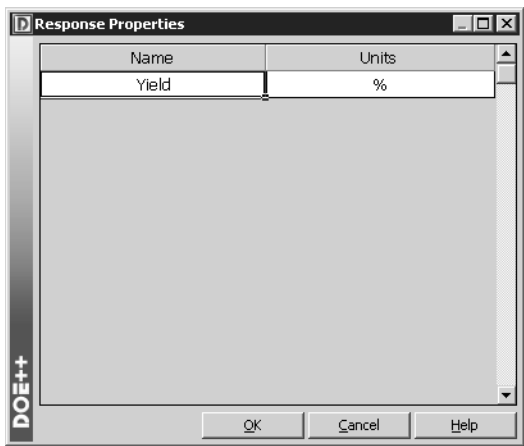

- In the third step of the Design Wizard, select **2** from the **Factors** drop-down and **5** from the **Center Points Per Block** drop-down and leave all other fields at their default settings, as shown next.



- Click the **Factor Properties** button. In the Factor Properties window that appears, enter the names, units and high and low level values for the factors, as shown next.

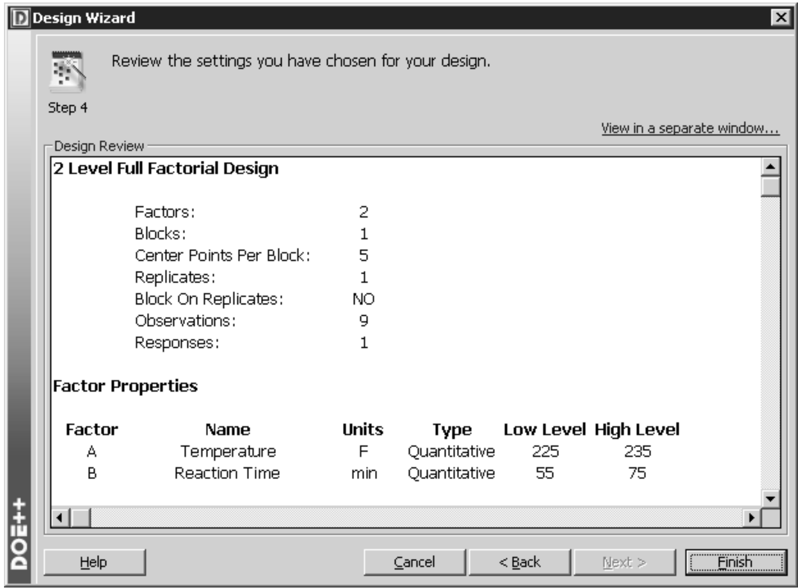



- Click **OK** to close the Factor Properties window and return to the Design Wizard.
- Click the **Response Properties** button. In the Response Properties window that appears, define the response name, as shown next.



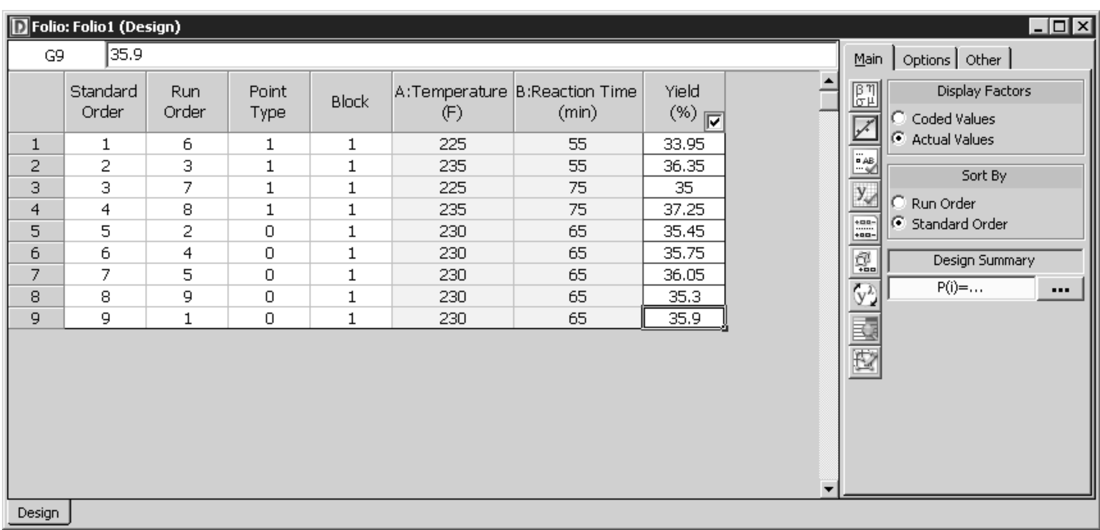

- Click **OK** to close the Response Properties window and return to the Design Wizard.
- Click **Next** to proceed to the next step.

- The fourth step of the Design Wizard provides a review of the settings you have chosen for the design.



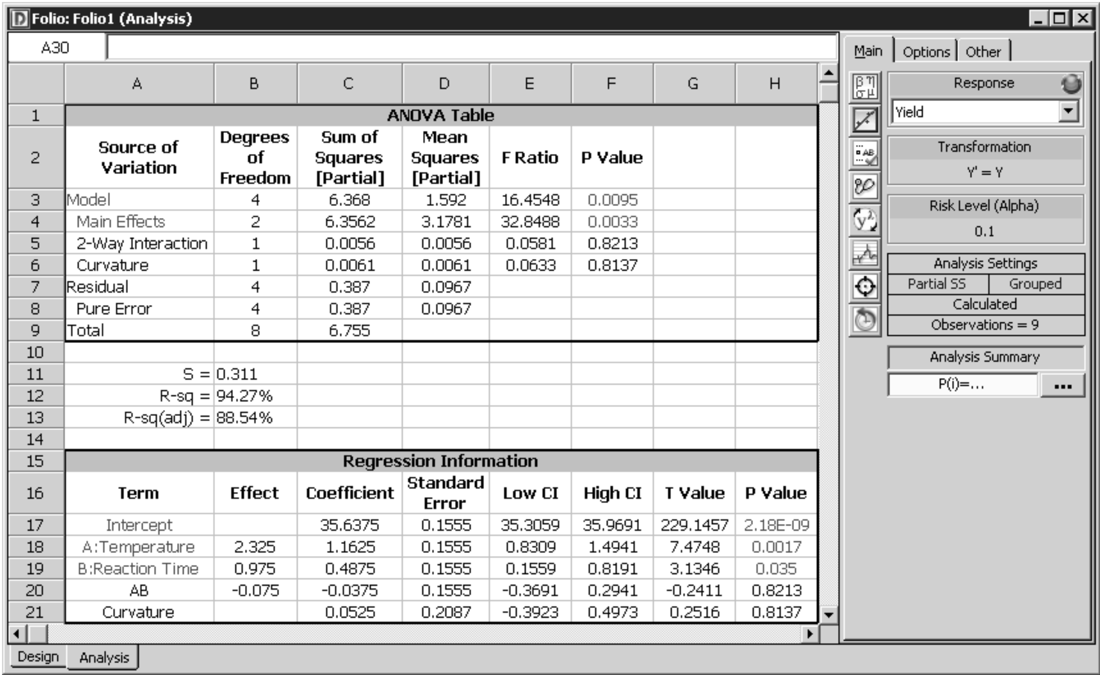

If any of the settings are not as intended, you can click the **Back** button to return to the appropriate step and make any necessary changes.

- Click **Finish** to create the new project with the Standard Folio containing the two level full factorial design.
- Save the project as “SequentialOptimization.rso7.”
- Select **Standard Order** in the **Sort By** area in the Control Panel to sort the Data Sheet by the Standard Order column, then enter the response data in the order given in this example on page 70. The Folio will look like the one shown next.




- On the Options page of the Control Panel, specify a **Risk Level (Alpha)** value of **0.1** for the Yield response and select to use **Grouped Terms**.

- Return to the Main page of the Control Panel and click **Calculate**. The Analysis tab is shown next.



| Source of Variation | Degrees of Freedom | Sum of Squares [Partial] | Mean Squares [Partial] | F Ratio | P Value |
|---|---|---|---|---|---|
| Model | 4 | 6.368 | 1.592 | 16.4548 | 0.0095 |
| Main Effects | 2 | 6.3562 | 3.1781 | 32.8488 | 0.0033 |
| 2-Way Interaction | 1 | 0.0056 | 0.0056 | 0.0581 | 0.8213 |
| Curvature | 1 | 0.0061 | 0.0061 | 0.0633 | 0.8137 |
| Residual | 4 | 0.387 | 0.0967 | | |
| Pure Error | 4 | 0.387 | 0.0967 | | |
| Total | 8 | 6.755 | | | |

S = 0.311
R-sq = 94.27%
R-sq(adj) = 88.54%

| Term | Effect | Coefficient | Standard Error | Low CI | High CI | T Value | P Value |
|---|---|---|---|---|---|---|---|
| Intercept | | 35.6375 | 0.1555 | 35.3059 | 35.9691 | 229.1457 | 2.18E-09 |
| A:Temperature | 2.325 | 1.1625 | 0.1555 | 0.8309 | 1.4941 | 7.4748 | 0.0017 |
| B:Reaction Time | 0.975 | 0.4875 | 0.1555 | 0.1559 | 0.8191 | 3.1346 | 0.035 |
| AB | -0.075 | -0.0375 | 0.1555 | -0.3691 | 0.2941 | -0.2411 | 0.8213 |
| Curvature | | 0.0525 | 0.2087 | -0.3923 | 0.4973 | 0.2516 | 0.8137 |



The analysis shows that the first-order regression model is adequate for this experiment since curvature is not significant. The interaction term is also not significant. Thus, dropping the interaction term, the model in terms of the coded variables is:

$$\hat{y} = 35.6375 + 1.1625x_1 + 0.4875x_2$$

## 4.7.2 Part 2: Method of Steepest Ascent

Do the following:

- Use the method of steepest ascent to arrive at the operating conditions that are close to the region of maximum yield.[7] Use a step size of 10 minutes for factor B.
- How would you check for the presence of curvature at this region?

**Solution**

The step size of 10 minutes in factor B (variable $x_2$) corresponds to a step size of 1 unit in coded values, as shown next:

$$\Delta x_2 coded = \frac{\Delta x_2 actual}{half\ range} = \frac{10}{\left(\frac{75-55}{2}\right)} = 1$$

7. For more information on the method of steepest ascent, please refer to *ReliaSoft's Experiment Design and Analysis Reference*.

In terms of coded values, that step size of 1 unit in $x_2$ corresponds to a step size of 2.39 units in $x_1$ (*i.e.* factor A):

$$\Delta x_1 \ coded = \frac{\hat{\beta}_1}{\hat{\beta}_1} \cdot \Delta x_2 \ coded = \frac{1.1625}{0.4875} \cdot 1 = 2.39$$

In terms of actual values, that step size of 2.39 coded units in $x_1$ corresponds to a step size of 11.95$^o$F:

$$\Delta x_1 \ actual = \Delta x_1 \ coded \cdot half \ range = 2.39 \cdot \frac{(235-225)}{2} = 11.95$$

Experiments should be conducted by increasing the temperature and the reaction time in steps of 12$^o$F and 10 minutes until no further increase is observed in the yield.

Following this protocol, the following information is observed at each step of the path of steepest ascent:

| **Steps** | **Factor Levels** | | | | **Yield** |
|---|---|---|---|---|---|
| | **Coded** | | **Actual** | | |
| | A | B | A | B | |
| Current Operation | 0 | 0 | 230 | 65 | |
| Delta | 2.4 | 1 | 12 | 10 | |
| 1 | 2.4 | 1 | 242 | 75 | 36.50 |
| 2 | 4.8 | 2 | 254 | 85 | 39.35 |
| 3 | 7.2 | 3 | 266 | 95 | 45.65 |
| 4 | 9.6 | 4 | 278 | 105 | 49.55 |
| 5 | 12.0 | 5 | 290 | 115 | 55.70 |
| 6 | 14.4 | 6 | 302 | 125 | 64.25 |
| 7 | 16.8 | 7 | 314 | 135 | 72.50 |
| 8 | 19.2 | 8 | 326 | 145 | 80.60 |
| 9 | 21.6 | 9 | 338 | 155 | 91.40 |
| **10** | **24.0** | **10** | **350** | **165** | **95.45** |
| 11 | 26.4 | 11 | 362 | 175 | 89.30 |
| 12 | 28.8 | 12 | 374 | 185 | 87.65 |

Settings in the region centered around $x_1$ = 350$^o$F and $x_2$ = 165 minutes (step 10) seem optimal, as the response is largest at these settings.

Using the method of steepest ascent, the region where yield is maximized was determined; however, the presence or absence of curvature in the region of interest cannot be determined. By using an experiment that is augmented with center points, as shown in the next section, we can further investigate for curvature.

### 4.7.3 Part 3: Checking for Curvature

In order to investigate the region that was determined in Part 2 to contain the maximum yield, a $2^2$ design augmented by 5 center points was conducted. The following data set was obtained:

| Center Point | Temperature (F) | Reaction Time (min) | Yield (%) |
|---|---|---|---|
| No | 345 | 155 | 89.75 |
| No | 355 | 155 | 90.20 |
| No | 345 | 175 | 92.00 |
| No | 355 | 175 | 94.25 |
| Yes | 350 | 165 | 94.85 |
| Yes | 350 | 165 | 95.45 |
| Yes | 350 | 165 | 95.00 |
| Yes | 350 | 165 | 94.55 |
| Yes | 350 | 165 | 94.70 |

Do the following:

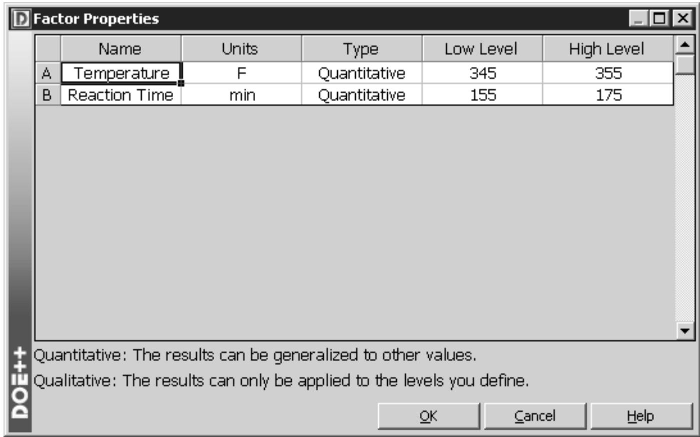
- Determine whether curvature is present.
- If curvature is present, what would be the simplest way to proceed in order to determine the optimum settings for temperature and reaction time?

*In the "Sequential Optimization.rdoe" sample file, the Folio for this part of the example is named "Folio2."*

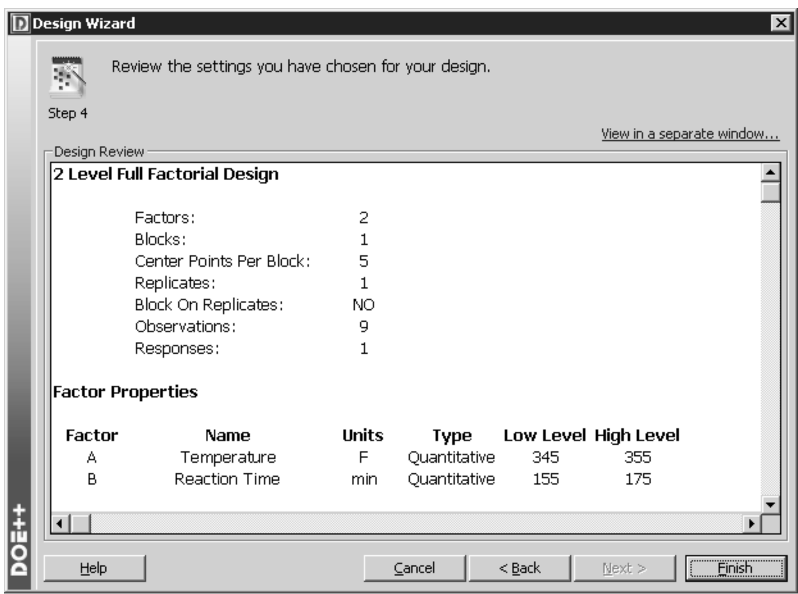
**Solution**

- Add a new Standard Folio to the project by selecting **Add Folio** from the **Project** menu. On the first page of the Design Wizard, click the **Load Last Design...** link. DOE++ will use all of the settings specified for the most recently created design, including factor and response properties, and go immediately to the design configuration step (in this case, the third step) of the Design Wizard.

- In the third step of the Design Wizard, click the **Factor Properties** button. In the Factor Properties window that appears, enter the new high and low level values for the factors, as shown next.



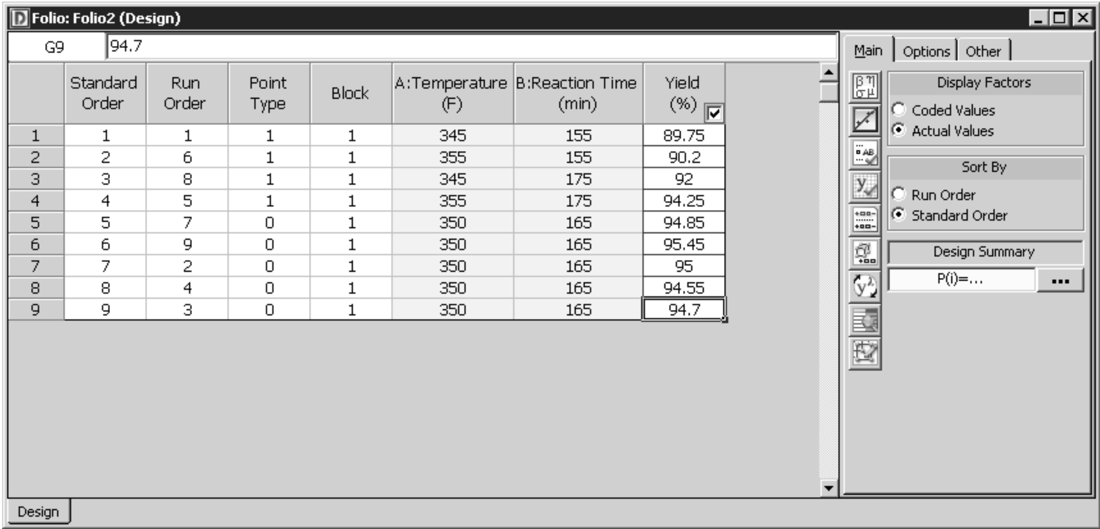

- Click **OK** to close the Factor Properties window and return to the Design Wizard.
- Click **Next** to proceed to the next step.
- The fourth step of the Design Wizard provides a review of the settings you have chosen for the design.




If any of the settings are not as intended, you can click the **Back** button to return to the appropriate step and make any necessary changes.

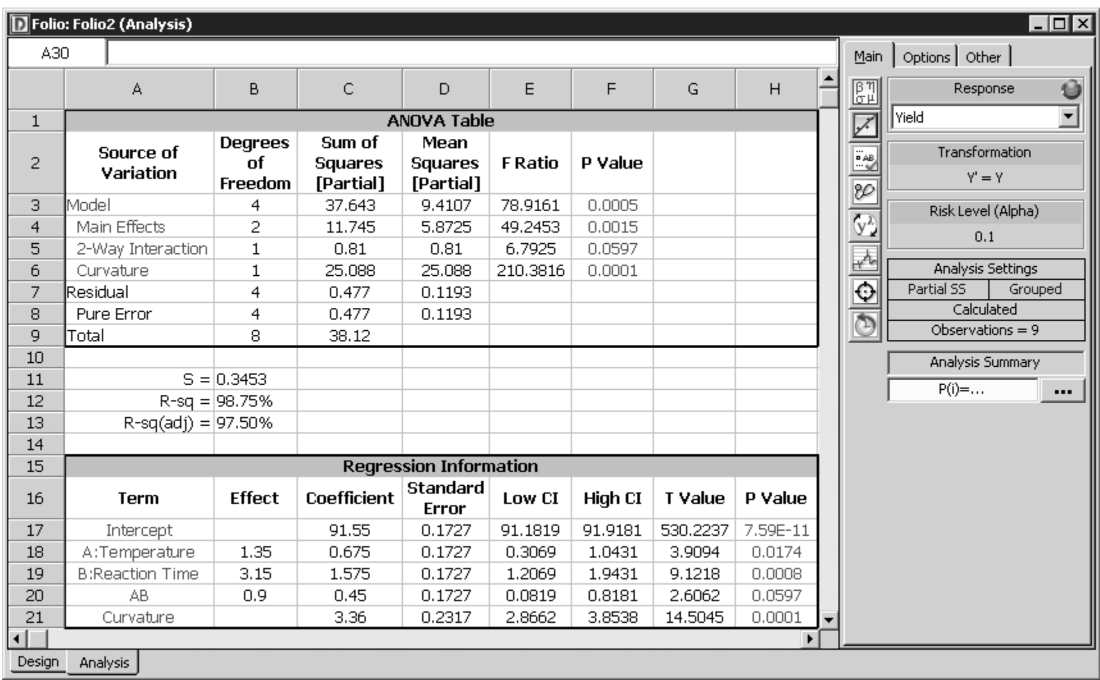
- Click **Finish** to create the new Standard Folio containing the two level full factorial design.

- Select **Standard Order** in the **Sort By** area in the Control Panel to sort the Data Sheet by the Standard Order column, then enter the response data in the order given in this example on page 77. The Folio will look like the one shown next.




| | Standard Order | Run Order | Point Type | Block | A:Temperature (F) | B:Reaction Time (min) | Yield (%) |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 345 | 155 | 89.75 |
| 2 | 2 | 6 | 1 | 1 | 355 | 155 | 90.2 |
| 3 | 3 | 8 | 1 | 1 | 345 | 175 | 92 |
| 4 | 4 | 5 | 1 | 1 | 355 | 175 | 94.25 |
| 5 | 5 | 7 | 0 | 1 | 350 | 165 | 94.85 |
| 6 | 6 | 9 | 0 | 1 | 350 | 165 | 95.45 |
| 7 | 7 | 2 | 0 | 1 | 350 | 165 | 95 |
| 8 | 8 | 4 | 0 | 1 | 350 | 165 | 94.55 |
| 9 | 9 | 3 | 0 | 1 | 350 | 165 | 94.7 |

- On the Options page of the Control Panel, specify a **Risk Level (Alpha)** value of **0.1** for the Yield response and select to use **Grouped Terms**.
- Return to the Main page of the Control Panel and click **Calculate**. The Analysis tab is shown next.




| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | ANOVA Table | | | | | | | |
| 2 | Source of Variation | Degrees of Freedom | Sum of Squares [Partial] | Mean Squares [Partial] | F Ratio | P Value | | |
| 3 | Model | 4 | 37.643 | 9.4107 | 78.9161 | 0.0005 | | |
| 4 | Main Effects | 2 | 11.745 | 5.8725 | 49.2453 | 0.0015 | | |
| 5 | 2-Way Interaction | 1 | 0.81 | 0.81 | 6.7925 | 0.0597 | | |
| 6 | Curvature | 1 | 25.088 | 25.088 | 210.3816 | 0.0001 | | |
| 7 | Residual | 4 | 0.477 | 0.1193 | | | | |
| 8 | Pure Error | 4 | 0.477 | 0.1193 | | | | |
| 9 | Total | 8 | 38.12 | | | | | |
| 10 | | | | | | | | |
| 11 | S = | 0.3453 | | | | | | |
| 12 | R-sq = | 98.75% | | | | | | |
| 13 | R-sq(adj) = | 97.50% | | | | | | |
| 14 | | | | | | | | |
| 15 | Regression Information | | | | | | | |
| 16 | Term | Effect | Coefficient | Standard Error | Low CI | High CI | T Value | P Value |
| 17 | Intercept | | 91.55 | 0.1727 | 91.1819 | 91.9181 | 530.2237 | 7.59E-11 |
| 18 | A:Temperature | 1.35 | 0.675 | 0.1727 | 0.3069 | 1.0431 | 3.9094 | 0.0174 |
| 19 | B:Reaction Time | 3.15 | 1.575 | 0.1727 | 1.2069 | 1.9431 | 9.1218 | 0.0008 |
| 20 | AB | 0.9 | 0.45 | 0.1727 | 0.0819 | 0.8181 | 2.6062 | 0.0597 |
| 21 | Curvature | | 3.36 | 0.2317 | 2.8662 | 3.8538 | 14.5045 | 0.0001 |

In the ANOVA table, the curvature and its $p$ value of the curvature are displayed in red, indicating that there is curvature that merits further investigation. The presence of curvature indicates that a linear model is insufficient for modeling the response of this experiment. Therefore, further investigation using an experiment that allows the use of second order models is required. The simplest way to perform this investigation and to determine the optimum settings for temperature and reaction time is to augment the current experiment with axial runs to complete a central composite design and fit a second order model to the response.

### 4.7.4 Part 4: RSM Analysis/Optimization

The following additional data points are obtained when performing the axial runs required for a response surface methodology (RSM) central composite design with $\alpha = 1.4142$ to get a rotatable design.

| Center Point | Temperature (F) | Reaction Time (min) | Yield (%) |
|---|---|---|---|
| No | 342.9289322 | 165 | 90.5 |
| No | 357.0710678 | 165 | 92.75 |
| No | 350 | 150.8578644 | 88.4 |
| No | 350 | 179.1421356 | 92.6 |

Do the following:

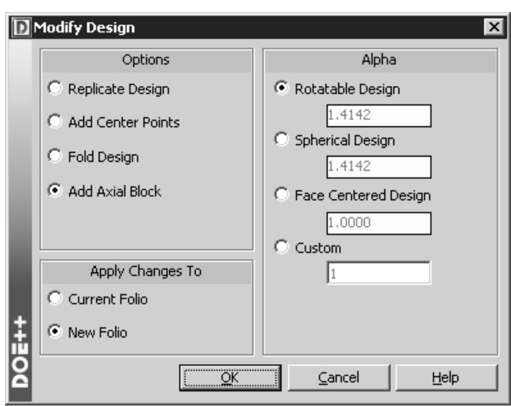
- Obtain the second order model using the significant terms.
- Find factor settings such that the yield is maximized.

*In the "Sequential Optimization.rdoe" sample file, the Folio for this part of the example is named "Folio3."*

**Solution:**

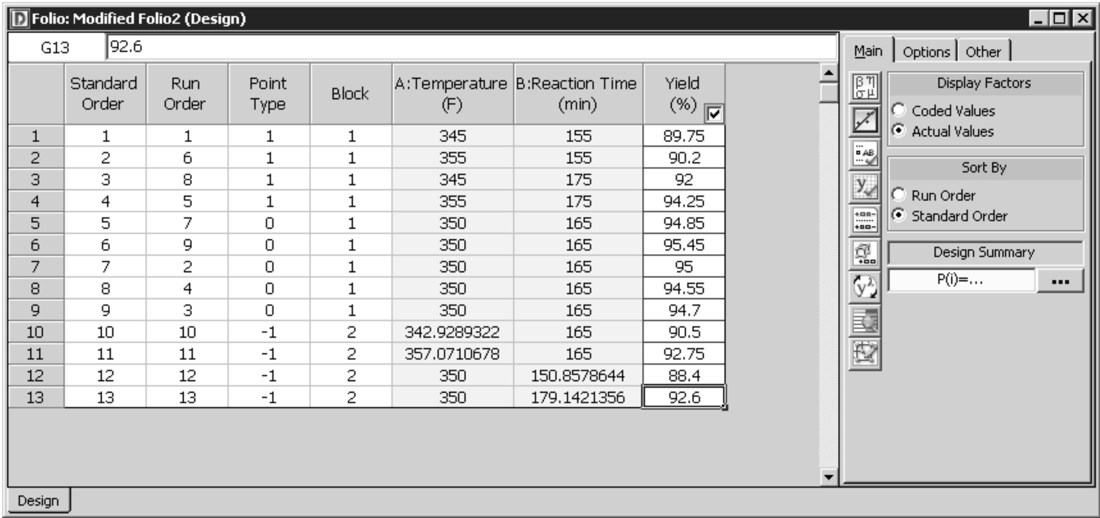
- Return to the Design tab of the second Standard Folio.
- Select **Modify Design** from the **Data** menu or click the **Modify Design** icon on the Control Panel.

- In the Modify Design window that appears, select **Add Axial Block** in the Options area. Select **New Folio** in the Apply Changes To area and select **Rotatable Design** for Alpha, as shown next, then click **OK**.

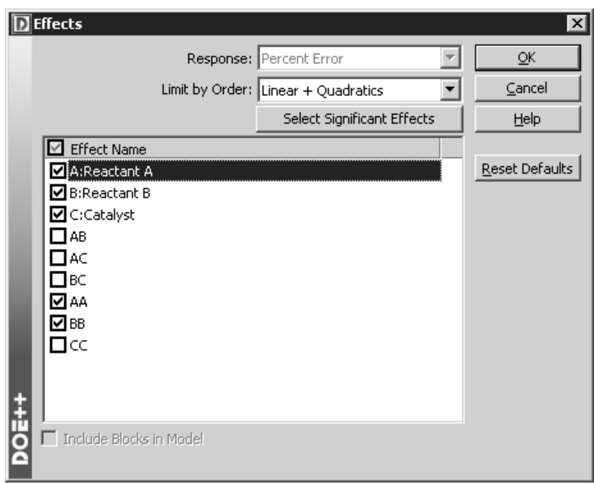



A new Standard Folio containing a copy of the design plus the settings for the axial points will be added to the project.

- Enter the axial point response data given in this example on page 80, as shown next.

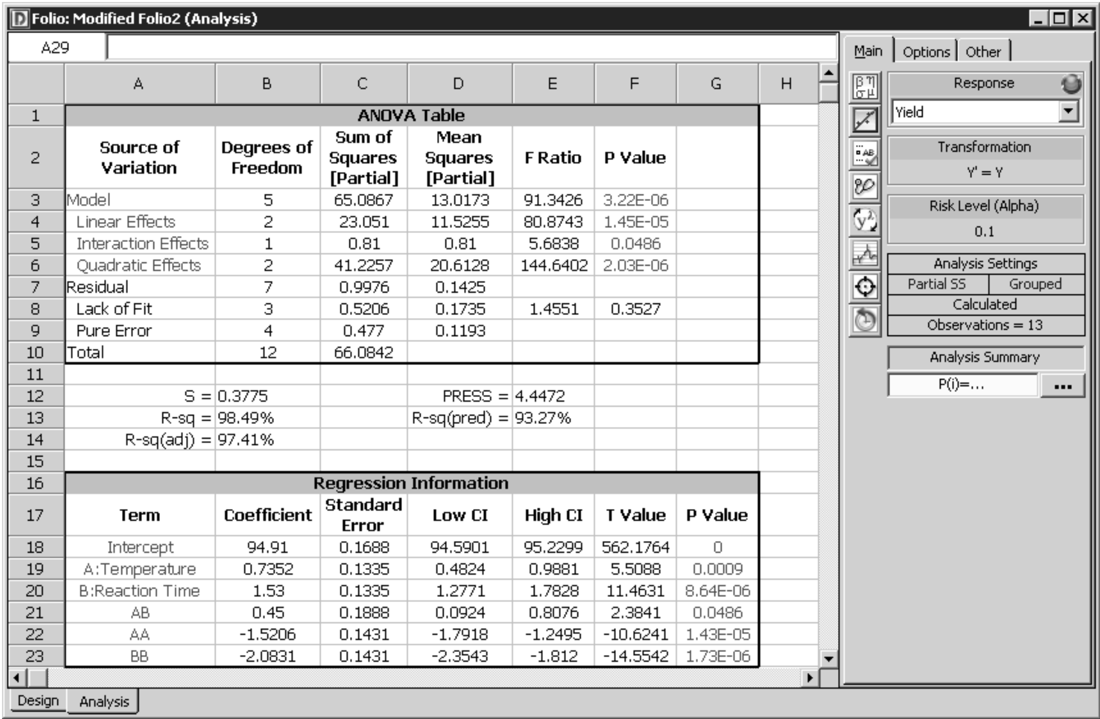



- Select **Select Effects** from the **Data** menu or click the **Select Effects** icon in the Control Panel.
- In the Effects window, clear the **Include Blocks in Model** option, as shown next.




- Click **OK** to accept your selections and close the window.
- On the Options page of the Control Panel, specify a **Risk Level (Alpha)** value of **0.1** for the Yield response and select to use **Grouped Terms**.

- Return to the Main page of the Control Panel and click **Calculate**. The Analysis tab is shown next.




| ANOVA Table | | | | | |
|---|---|---|---|---|---|
| Source of Variation | Degrees of Freedom | Sum of Squares [Partial] | Mean Squares [Partial] | F Ratio | P Value |
| Model | 5 | 65.0867 | 13.0173 | 91.3426 | 3.22E-06 |
| Linear Effects | 2 | 23.051 | 11.5255 | 80.8743 | 1.45E-05 |
| Interaction Effects | 1 | 0.81 | 0.81 | 5.6838 | 0.0486 |
| Quadratic Effects | 2 | 41.2257 | 20.6128 | 144.6402 | 2.03E-06 |
| Residual | 7 | 0.9976 | 0.1425 | | |
| Lack of Fit | 3 | 0.5206 | 0.1735 | 1.4551 | 0.3527 |
| Pure Error | 4 | 0.477 | 0.1193 | | |
| Total | 12 | 66.0842 | | | |

| | | | |
|---|---|---|---|
| S = | 0.3775 | PRESS = | 4.4472 |
| R-sq = | 98.49% | R-sq(pred) = | 93.27% |
| R-sq(adj) = | 97.41% | | |

| Regression Information | | | | | | |
|---|---|---|---|---|---|---|
| Term | Coefficient | Standard Error | Low CI | High CI | T Value | P Value |
| Intercept | 94.91 | 0.1688 | 94.5901 | 95.2299 | 562.1764 | 0 |
| A:Temperature | 0.7352 | 0.1335 | 0.4824 | 0.9881 | 5.5088 | 0.0009 |
| B:Reaction Time | 1.53 | 0.1335 | 1.2771 | 1.7828 | 11.4631 | 8.64E-06 |
| AB | 0.45 | 0.1888 | 0.0924 | 0.8076 | 2.3841 | 0.0486 |
| AA | -1.5206 | 0.1431 | -1.7918 | -1.2495 | -10.6241 | 1.43E-05 |
| BB | -2.0831 | 0.1431 | -2.3543 | -1.812 | -14.5542 | 1.73E-06 |

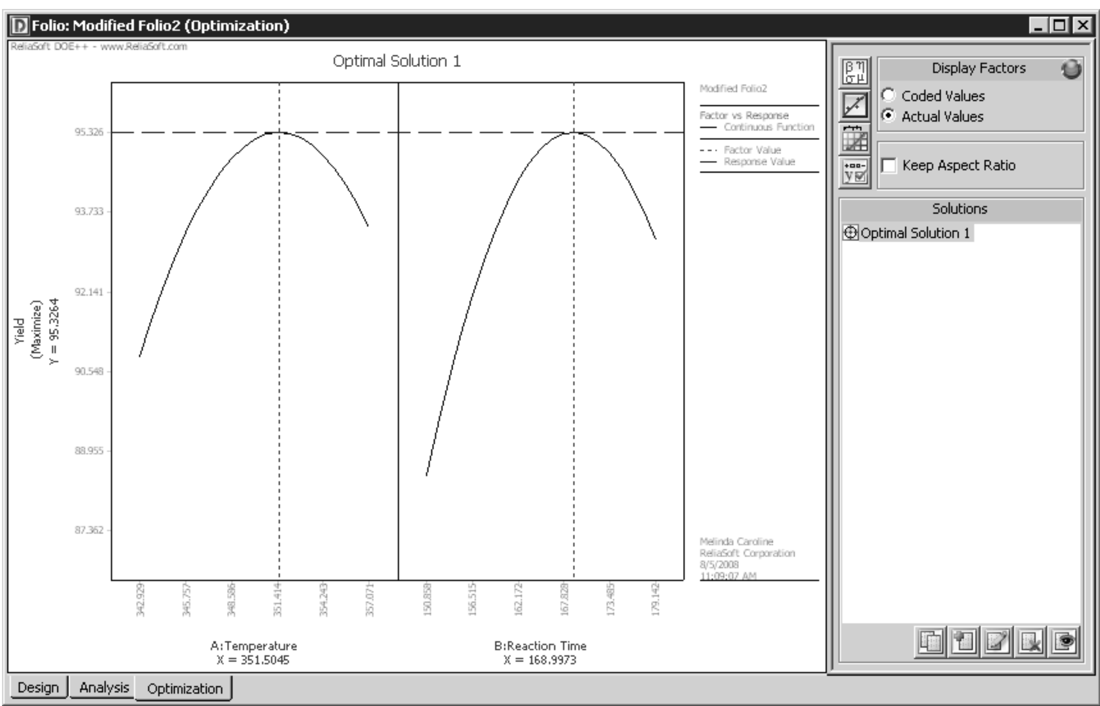
All effects are significant at a significance level of 0.1. The final model in term of coded values is:

$$\hat{y} = 94.9 + 0.735x_1 + 1.53x_1 + 0.45x_1x_2 - 1.52x_1^2 - 2.08x_2^2$$

- Select **Optimization** from the **Data** menu or click the **Optimization** icon in the Control Panel.

- In the Optimization Settings window that appears, select **Maximize** from the drop-down in the Goal column and enter **80** for the Lower Limit value and **100** for the Target value. The Optimization Settings window will look like the one shown next.


- Click **OK** to accept your settings and create the Optimal Solution plot, as shown next.




The optimum settings are found to be at a temperature of $x_1 = 352^oF$ and with a reaction time of $x_2 = 169$ minutes.

- Save and close the project then proceed to the next example.

## 4.8 Example 8: Taguchi Robust Design

Consider the case of a chemical process where the experimenter wants the product to be neither acidic nor basic (*i.e*. the pH of the product needs to be as close to 7 as possible). It is thought that the pH of the product depends on the concentration of the three reactants A, B and C used to create the product. It has also been found that the pH of the product depends on the ambient temperature.

The experimenter chooses Taguchi's robust parameter design approach to investigate the settings of the control factors (*i.e.* the concentrations of A, B and C) to make the product as insensitive as possible to the noise factor (*i.e.* changes in ambient temperature). It is decided to carry out a $2^3$ experiment to study the effect of the three control factors on the pH of the product. It is also decided to carry out the experiment at four levels of the ambient temperature by using a chamber where the surrounding temperature of the chemical process can be controlled.

The resulting data set is shown next.

| Control Factor Settings | | | Noise Condition 1 | Noise Condition 2 | Noise Condition 3 | Noise Condition 4 |
|---|---|---|---|---|---|---|
| A | B | C | | | | |
| -1 | -1 | -1 | 5.05 | 6.5 | 7 | 7.15 |
| 1 | -1 | -1 | 5.05 | 5.05 | 5.35 | 6.15 |
| -1 | 1 | -1 | 5.95 | 6.85 | 7.65 | 7.95 |
| 1 | 1 | -1 | 4.65 | 4.95 | 5.35 | 5 |
| -1 | -1 | 1 | 6.35 | 7 | 7.25 | 7 |
| 1 | -1 | 1 | 4.05 | 6.55 | 6.15 | 6.45 |
| -1 | 1 | 1 | 6.85 | 7.05 | 7 | 8 |
| 1 | 1 | 1 | 4.65 | 5.55 | 6 | 6.15 |

Note that the factor values in this data set are expressed as coded values, where -1 represents the factor's low value and 1 represents the factor's high value. When creating a design, with the exception of standard one factor designs, it is not necessary to enter actual factor values. In some situations, it may be more convenient to use coded values. DOE++ performs analysis in the same way, regardless of whether actual or coded values are used.

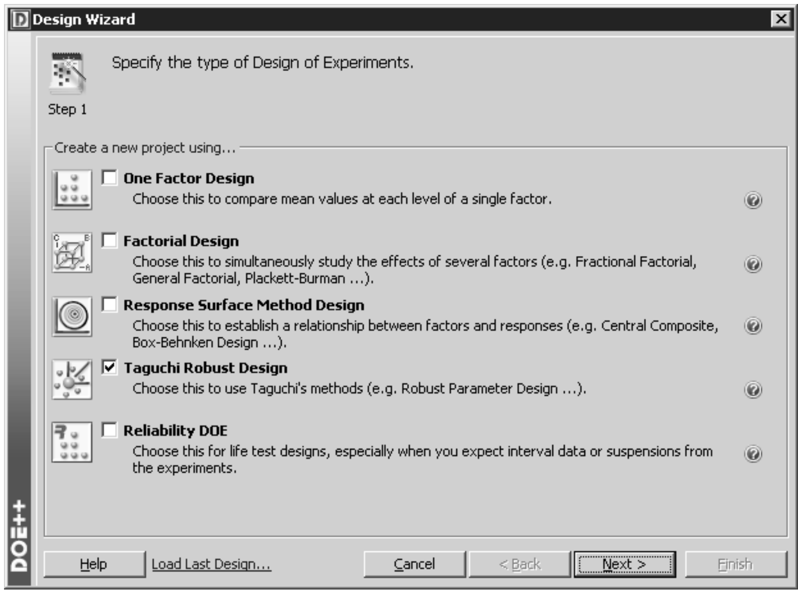
Do the following:

- Obtain the location and dispersion models using a nominal-the-best model for the Signal/Noise Ratio model.
- Using the normal probability plots of the location and signal-to-noise ratio responses, identify the significant effects at a 10% significance level.
- Build location and signal-to-noise ratio models.
- Find the optimum settings so that the pH value stays on target while maximizing the signal-to-noise ratio.

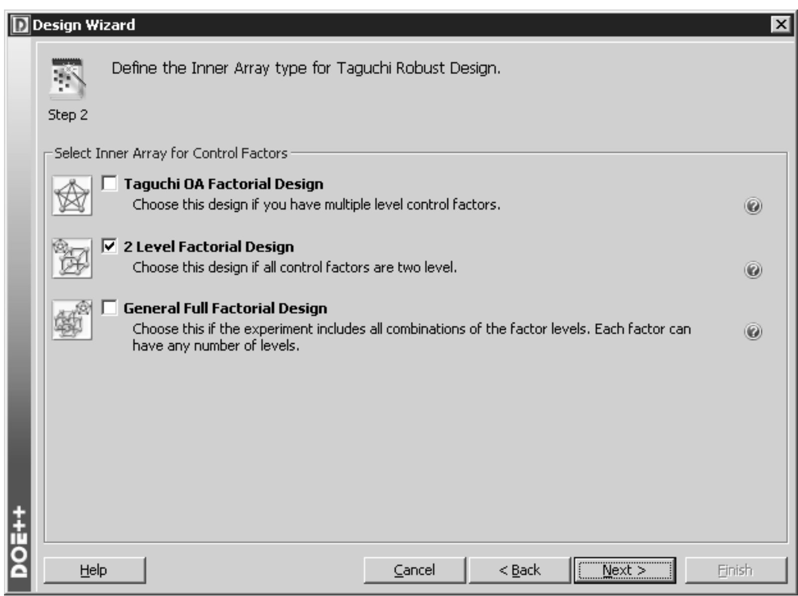
*The sample file for this example is located in the "Training Guide" folder in your application directory (e.g. C:\Program Files\ReliaSoft\DOE\Training Guide) and is named "TaguchiRobust.rdoe." The data set for* *this example is available in the Spreadsheet in the "TaguchiRobust-Data.rso7" file located in the "Raw Data" folder within the "Training Guide" folder.*

**Solution**

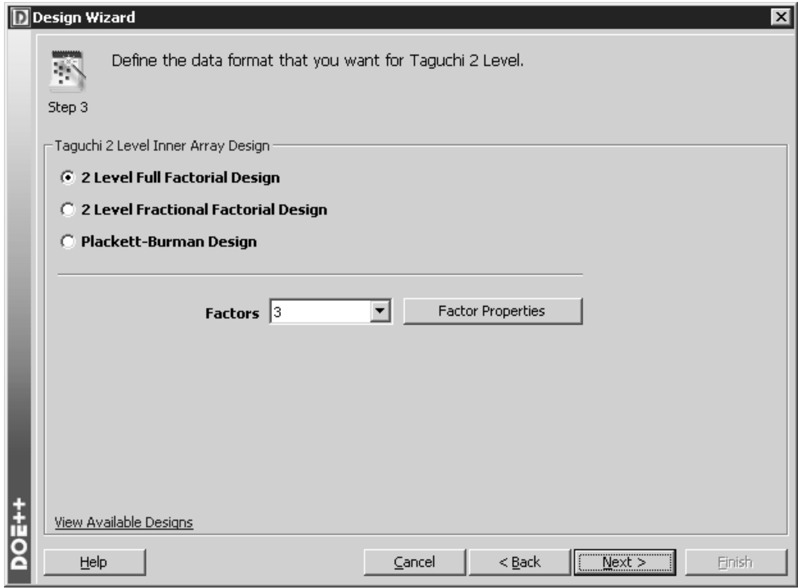
- Create a new project. On the first page of the Design Wizard, select to create a **Taguchi Robust Design** and click **Next** to proceed to the next step.



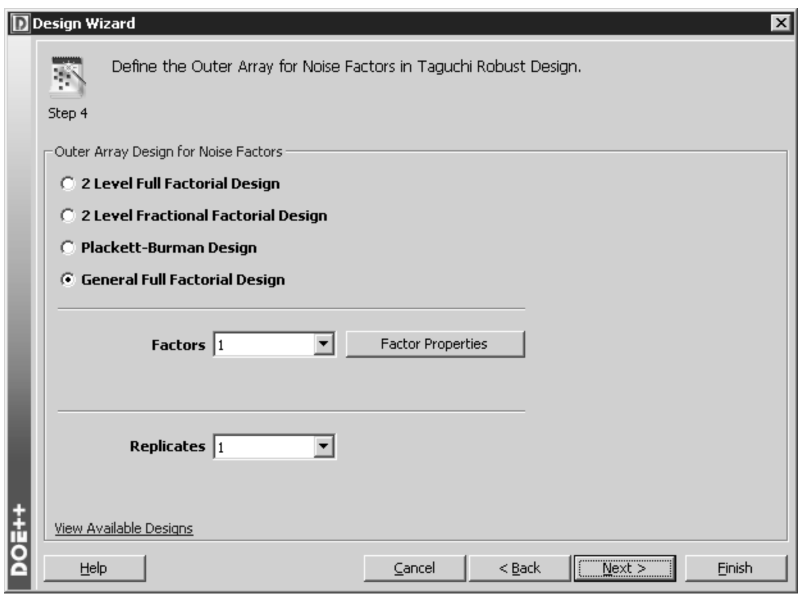

- In the second step of the Design Wizard, select **2 Level Factorial Design** for the inner (control) array type and click **Next** to proceed to the next step.



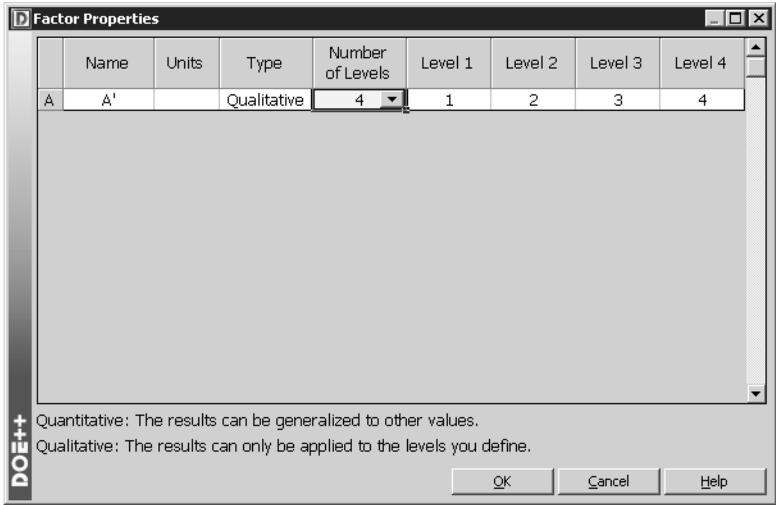

- In the third step of the Design Wizard, select to create a **2 Level Full Factorial Design** for the inner array and select **3** from the **Factors** drop-down and leave all other fields at their default settings. Because the control factor names in this example are A, B and C and only coded values are used, you do not need to change the factor names or enter low and high values for the factors. There is no need, therefore, to access the Factor Properties window. Click **Next** to proceed to the next step.



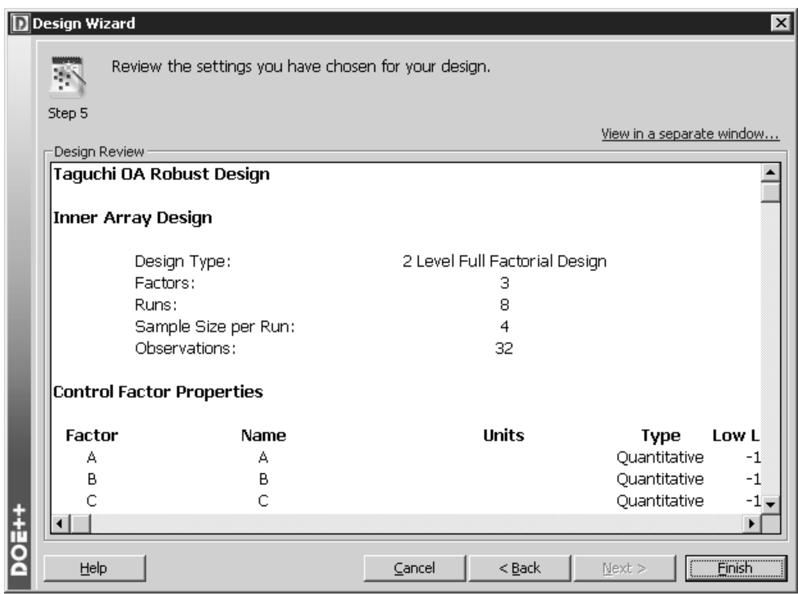

- In the fourth step of the Design Wizard, select to create a **General Full Factorial Design** for the outer (noise) array and leave all other fields at their default settings, as shown next.



- To define the noise factor properties, click the **Factor Properties** button. In the Factor Properties window that appears, select **4** from the drop-down in the **Number of Levels** column, as shown next.



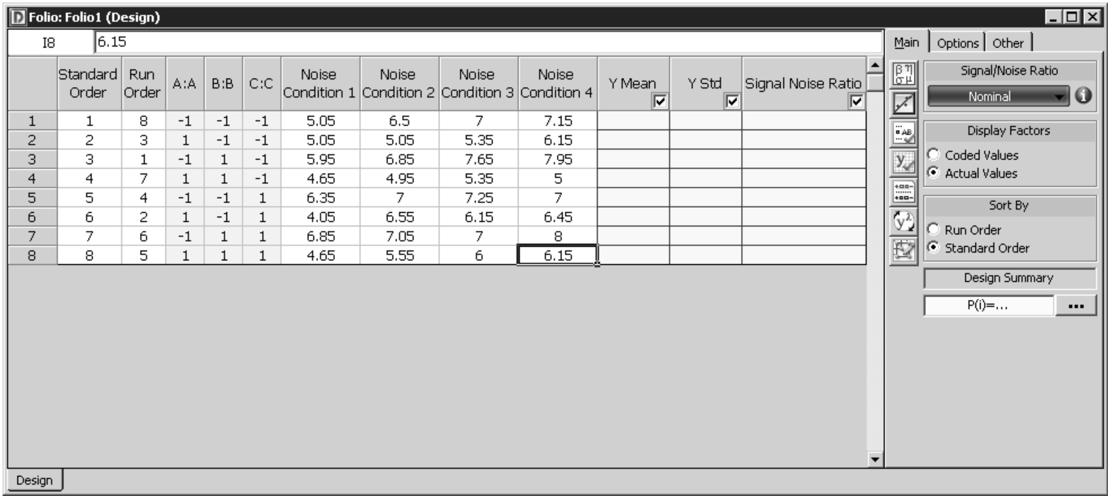

- Click **OK** to close the Factor Properties window and return to the Design Wizard.
- Click **Next** to proceed to the next step.
- The fifth step of the Design Wizard provides a review of the settings you have chosen for the design.




The information here is extensive. You may wish to take a moment to scroll through it. If any of the settings are not as intended, you can click the **Back** button to return to the appropriate step and make any necessary changes.

- Click **Finish** to create the new project with the Standard Folio containing the Taguchi robust design.
- Save the project as “TaguchiRobust.rso7.”

- Select **Standard Order** in the **Sort By** area in the Control Panel to sort the Data Sheet by the Standard Order column, then enter the response data for each noise condition in the order given in this example on page 84. The Folio will look like the one shown next.

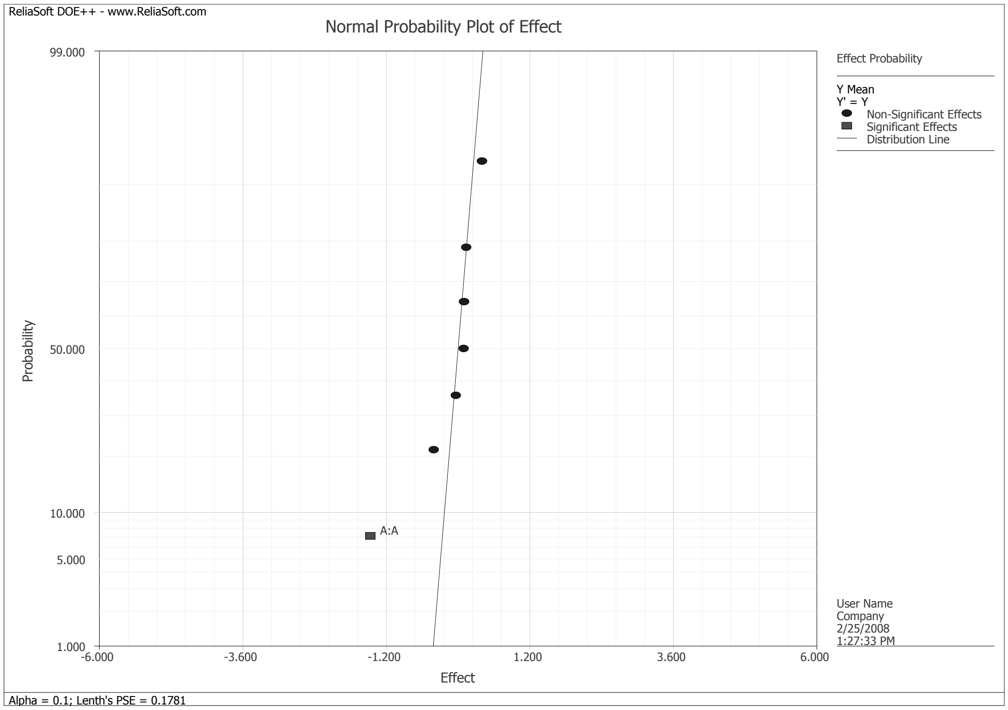



| | Standard Order | Run Order | A:A | B:B | C:C | Noise Condition 1 | Noise Condition 2 | Noise Condition 3 | Noise Condition 4 | Y Mean | Y Std | Signal Noise Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 8 | -1 | -1 | -1 | 5.05 | 6.5 | 7 | 7.15 | | | |
| 2 | 2 | 3 | 1 | -1 | -1 | 5.05 | 5.05 | 5.35 | 6.15 | | | |
| 3 | 3 | 1 | -1 | 1 | -1 | 5.95 | 6.85 | 7.65 | 7.95 | | | |
| 4 | 4 | 7 | 1 | 1 | -1 | 4.65 | 4.95 | 5.35 | 5 | | | |
| 5 | 5 | 4 | -1 | -1 | 1 | 6.35 | 7 | 7.25 | 7 | | | |
| 6 | 6 | 2 | 1 | -1 | 1 | 4.05 | 6.55 | 6.15 | 6.45 | | | |
| 7 | 7 | 6 | -1 | 1 | 1 | 6.85 | 7.05 | 7 | 8 | | | |
| 8 | 8 | 5 | 1 | 1 | 1 | 4.65 | 5.55 | 6 | 6.15 | | | |

Because you did not change the factor values from the original coded values in the Factor Properties window, the Folio will look the same regardless of whether **Coded Values** or **Actual Values** is selected in the **Display Factors** area of the Control Panel. Note that the last three columns, Y Mean, Y Std and Signal Noise Ratio, are currently empty. These columns will be populated by DOE++ when the data set is analyzed, using calculations based on your input in the noise condition columns, and will then be used as the responses for the analysis.

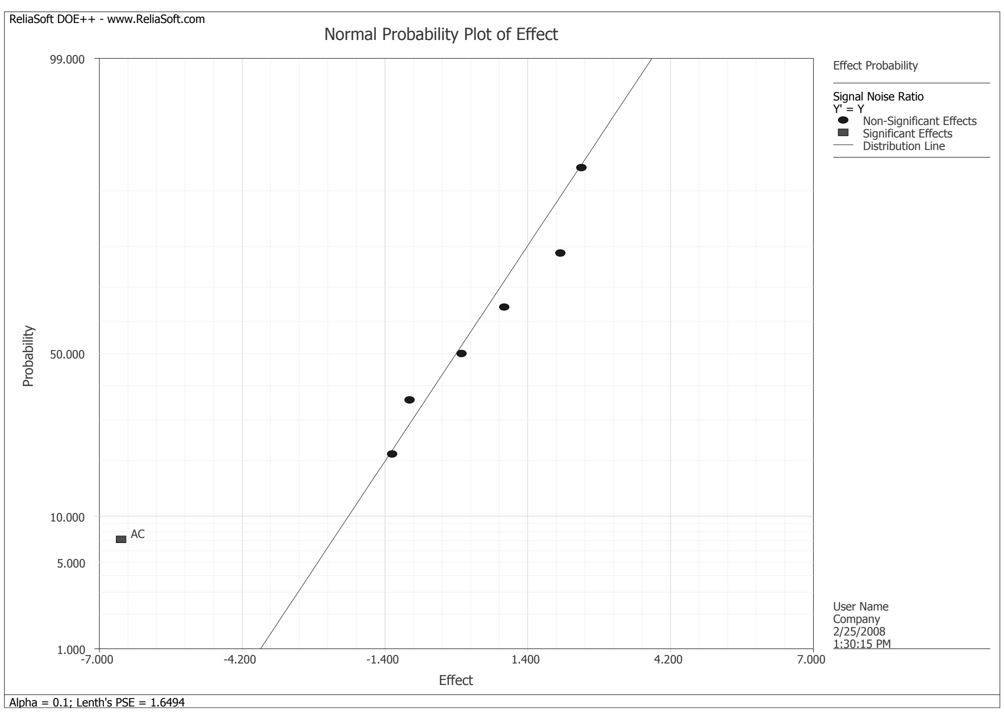
- On the Main page of the Control Panel, select **Nominal** in the **Signal/Noise Ratio** menu.
- On the Options page of the Control Panel, specify a **Risk Level (Alpha)** value of **0.1** and select to use **Grouped Terms** for the Y Mean response, which is currently selected in the **Response** drop-down.
- Select the **Y Std** response from the **Response** drop-down and specify a **Risk Level (Alpha)** value of **0.1** and select to use **Grouped Terms** for it.
- Do the same for the **Signal Noise Ratio** response.

- Return to the Main page of the Control Panel and click **Plot**. The Y Mean, Y Std and Signal Noise Ratio columns will be calculated and used as the responses in the Analysis and Plot tabs that are added to the Folio. The Effect Probability plot for the Y Mean (location) response, created by default, is shown next.




The normal probability plot of the effects for the location response indicates that A is the only significant effect for this model.

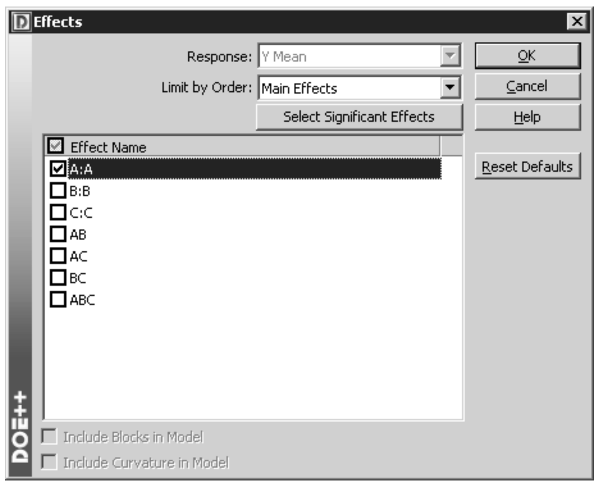
- Select **Signal Noise Ratio** from the **Response** drop-down in the Control Panel. The Effect Probability plot for the Signal Noise Ratio response is shown next.



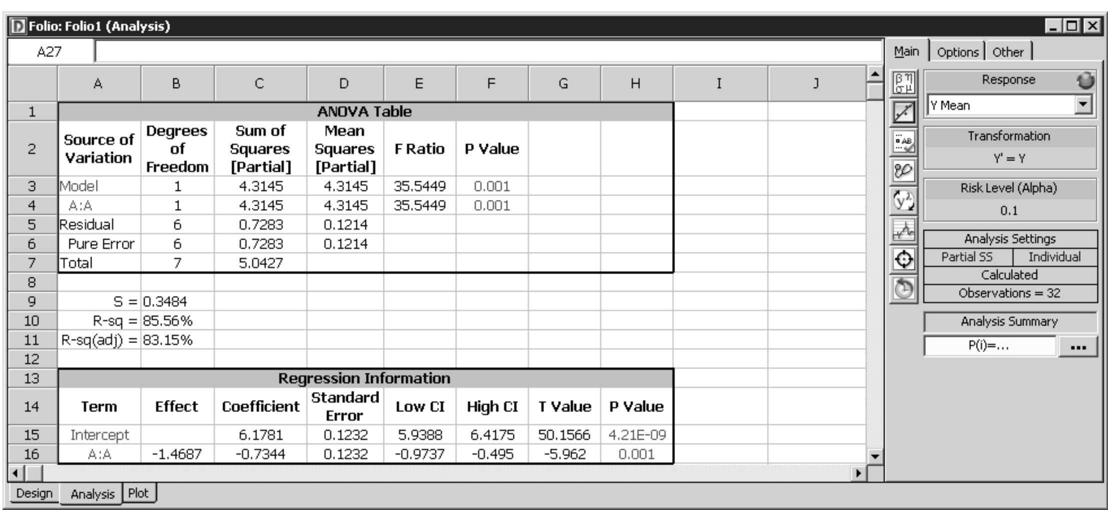
The normal probability plot of the effects for the Signal Noise Ratio response indicates that AC is the only significant effect for this model.

- Go to the Analysis tab by clicking its page index tab.
- Select the **Y Mean** response from the **Response** drop-down in the Control Panel if it is not already selected.
- Select **Select Effects** from the **Data** menu or click the **Select Effects** icon in the Control Panel.

- In the Effects window, click the **Select Significant Effects** button to select only the effect (**A**) shown to be significant for the Y Mean response, as shown next.




- Click **OK** to accept your selections and close the window.
- In the Analysis Settings area of the Control Panel, click inside the Test Terms area to toggle the analysis settings from **Grouped** to **Individual**.
- Click the **Calculate** icon to re-analyze the data set using the selections you have just made. The changed Analysis tab is shown next.



ANOVA Table

| Source of Variation | Degrees of Freedom | Sum of Squares [Partial] | Mean Squares [Partial] | F Ratio | P Value |
|---|---|---|---|---|---|
| Model | 1 | 4.3145 | 4.3145 | 35.5449 | 0.001 |
| A:A | 1 | 4.3145 | 4.3145 | 35.5449 | 0.001 |
| Residual | 6 | 0.7283 | 0.1214 | | |
| Pure Error | 6 | 0.7283 | 0.1214 | | |
| Total | 7 | 5.0427 | | | |

S = 0.3484
R-sq = 85.56%
R-sq(adj) = 83.15%

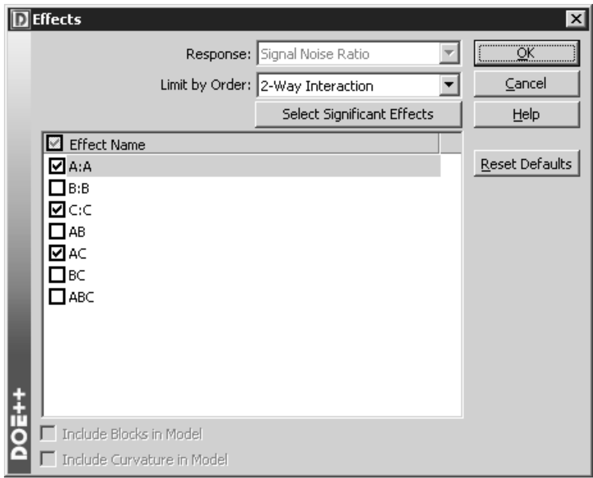
Regression Information

| Term | Effect | Coefficient | Standard Error | Low CI | High CI | T Value | P Value |
|---|---|---|---|---|---|---|---|
| Intercept | | 6.1781 | 0.1232 | 5.9388 | 6.4175 | 50.1566 | 4.21E-09 |
| A:A | -1.4687 | -0.7344 | 0.1232 | -0.9737 | -0.495 | -5.962 | 0.001 |

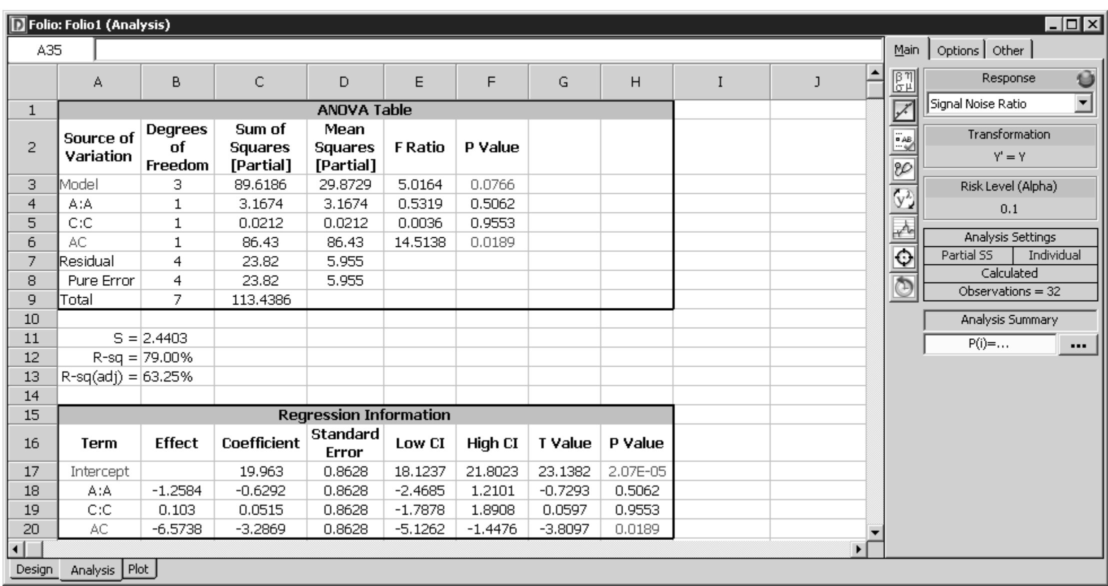
From the information in the Regression Information table, the location model can be determined to be:

$$\bar{y} = 6.1781 - 0.7344x_1$$

- Select the **Signal Noise Ratio** response from the **Response** drop-down in the Control Panel.
- Select **Select Effects** from the **Data** menu or click the **Select Effects** icon in the Control Panel.
- In the Effects window, click the **Select Significant Effects** button to select only the effects shown to be significant for the Signal Noise Ratio response. Note that in order to select **AC**, **A** and **C** must also be selected so that the model is hierarchical, as shown next.




- Click **OK** to accept your selections and close the window.
- In the Analysis Settings area of the Control Panel, click inside the Test Terms area to toggle the analysis settings from **Grouped** to **Individual**.
- Click the **Calculate** icon to re-analyze the data set using the selections you have just made. The changed Analysis tab is shown next.




ANOVA Table

| Source of Variation | Degrees of Freedom | Sum of Squares [Partial] | Mean Squares [Partial] | F Ratio | P Value |
|---|---|---|---|---|---|
| Model | 3 | 89.6186 | 29.8729 | 5.0164 | 0.0766 |
| A:A | 1 | 3.1674 | 3.1674 | 0.5319 | 0.5062 |
| C:C | 1 | 0.0212 | 0.0212 | 0.0036 | 0.9553 |
| AC | 1 | 86.43 | 86.43 | 14.5138 | 0.0189 |
| Residual | 4 | 23.82 | 5.955 | | |
| Pure Error | 4 | 23.82 | 5.955 | | |
| Total | 7 | 113.4386 | | | |

S = 2.4403
R-sq = 79.00%
R-sq(adj) = 63.25%

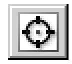
Regression Information

| Term | Effect | Coefficient | Standard Error | Low CI | High CI | T Value | P Value |
|---|---|---|---|---|---|---|---|
| Intercept | | 19.963 | 0.8628 | 18.1237 | 21.8023 | 23.1382 | 2.07E-05 |
| A:A | -1.2584 | -0.6292 | 0.8628 | -2.4685 | 1.2101 | -0.7293 | 0.5062 |
| C:C | 0.103 | 0.0515 | 0.8628 | -1.7878 | 1.8908 | 0.0597 | 0.9553 |
| AC | -6.5738 | -3.2869 | 0.8628 | -5.1262 | -1.4476 | -3.8097 | 0.0189 |

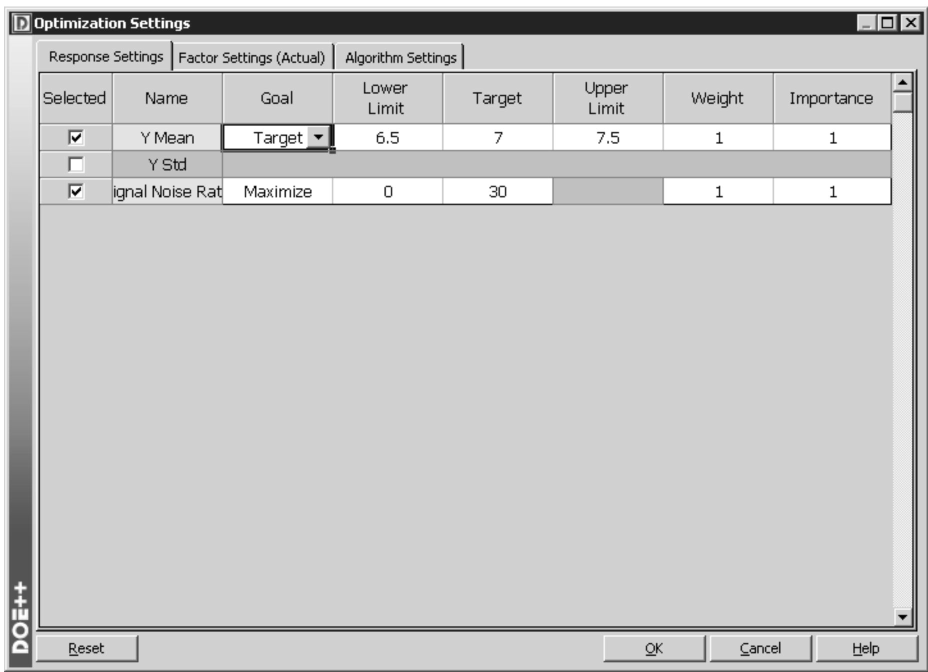
From the information in the Regression Information table, the dispersion model can be determined to be:

$$SN_T = 19.963 - 0.6292x_1 + 0.0515x_3 - 3.2869x_1x_3$$

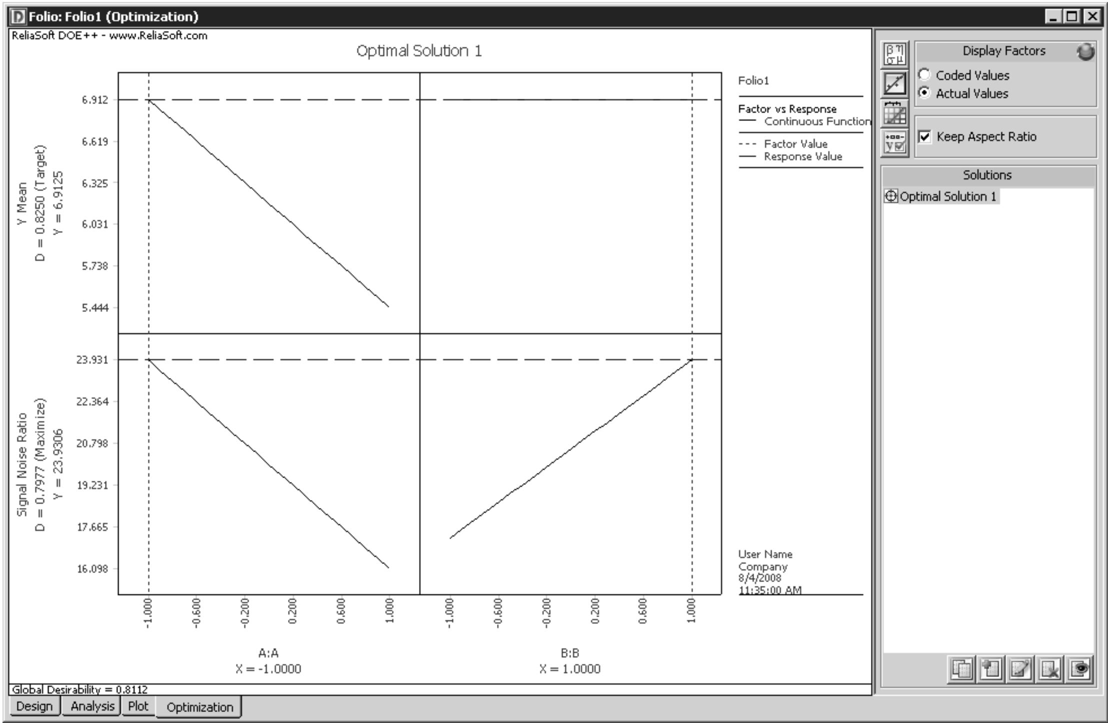
- Select **Optimization** from the **Data** menu or click the **Optimization** icon in the Control Panel.



- In the Optimization Settings window that appears, select the checkboxes for the **Y Mean** and **Signal Noise Ratio** responses and clear the checkbox for the **Y Std** response. For the Y Mean response, select **Target** from the drop-down in the Goal column and enter **6.5** for the Lower Limit value, **7** for the Target value and **7.5** for the Upper Limit value. For the Signal Noise Ratio response, select **Maximize** in the Goal column and enter **0** for the Lower Limit value and **30** for the Target value. The Optimization Settings window will look like the one shown next.



- Click **OK** to accept your settings and create the Optimal Solution plot, as shown next.




Based on the Optimal Solution plot, factor A should be set at its low value and factor C should be set at its high value in order to keep the pH as close as possible to 7 while making the product as insensitive as possible to changes in ambient temperature. Since factor B does not affect either the location model or the dispersion model, it can be set at the level that is most economical.

- Save and close the project then proceed to the next example.

## 4.9 Example 9: One Factor Reliability Design

A unit has traditionally worked in an application where it is required to cyclically bend to 40 degrees. The designers have been requested to inform management how it will affect the life of the parts to be working in an environment where the units will be required to bend as much as 50 to 60 degrees. Thirty units are to be randomly tested to failure by bending them to 40, 50 and 60 degrees and the cycles to failure will be recorded.

The following failure data are collected:

| Angle (degrees) | Cycles Failed |
|---|---|
| 40 | 75 |
| 50 | 33 |
| 60 | 35 |
| 40 | 53 |
| 50 | 35 |

| Angle (degrees) | Cycles Failed |
|---|---|
| 60 | 45 |
| 40 | 44 |
| 50 | 32 |
| 60 | 30 |
| 40 | 43 |
| 50 | 47 |
| 60 | 36 |
| 40 | 51 |
| 50 | 35 |
| 60 | 28 |
| 40 | 58 |
| 50 | 45 |
| 60 | 32 |
| 40 | 67 |
| 50 | 50 |
| 60 | 39 |
| 40 | 69 |
| 50 | 43 |
| 60 | 37 |
| 40 | 76 |
| 50 | 48 |
| 60 | 42 |
| 40 | 41 |
| 50 | 31 |
| 60 | 38 |

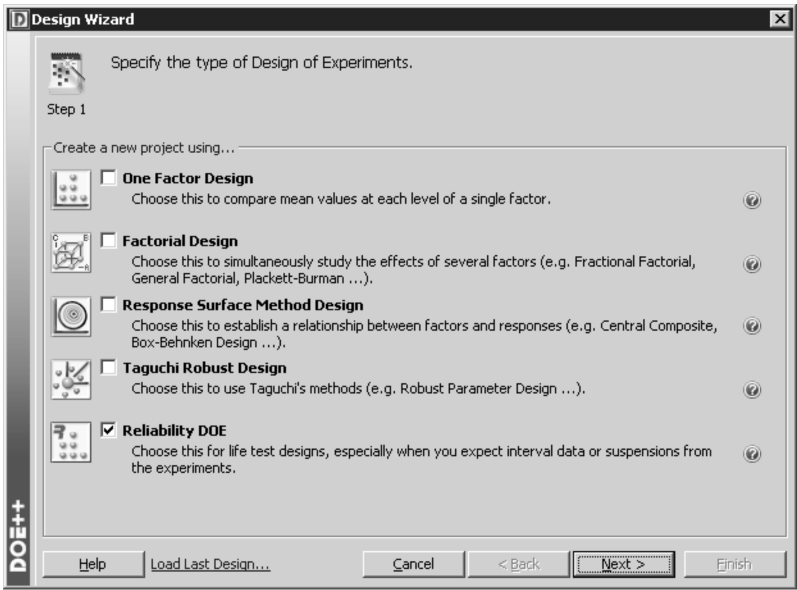
Do the following:

- Assuming a Weibull distribution and a significance level of 10%, determine whether the angle significantly affects the life of these units.
- Graphically validate the normality assumption.

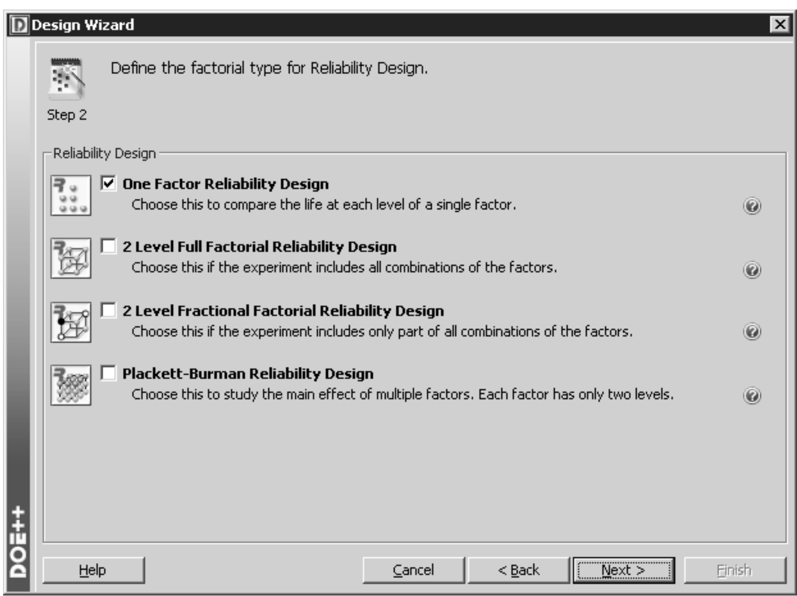
*The sample file for this example is located in the "Training Guide" folder in your application directory (e.g. C:\Program Files\ReliaSoft\DOE\Training Guide) and is named "OneFactorReliability.rdoe." The data set* *for this example is available in the Spreadsheet in the "OneFactorReliability-Data.rso7" file located in the "Raw Data" folder within the "Training Guide" folder.*

**Solution**

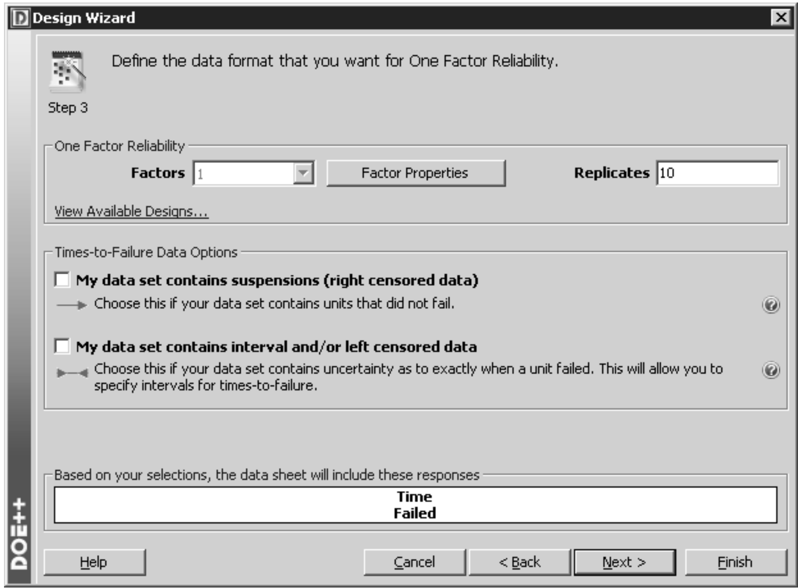
- Create a new project. On the first page of the Design Wizard, select to create a **Reliability DOE** and click **Next** to proceed to the next step.



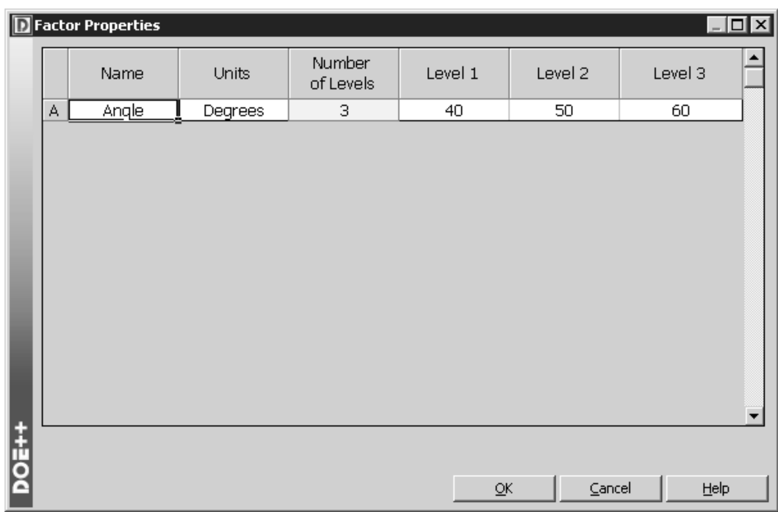

- In the second step of the Design Wizard, select **One Factor Reliability Design** and click **Next** to proceed to the next step.



- In the third step of the Design Wizard, enter **10** in the **Replicates** field, as each angle is tested 10 times. Leave all other fields at their default settings, indicating that there is no censoring in the data, as shown next.



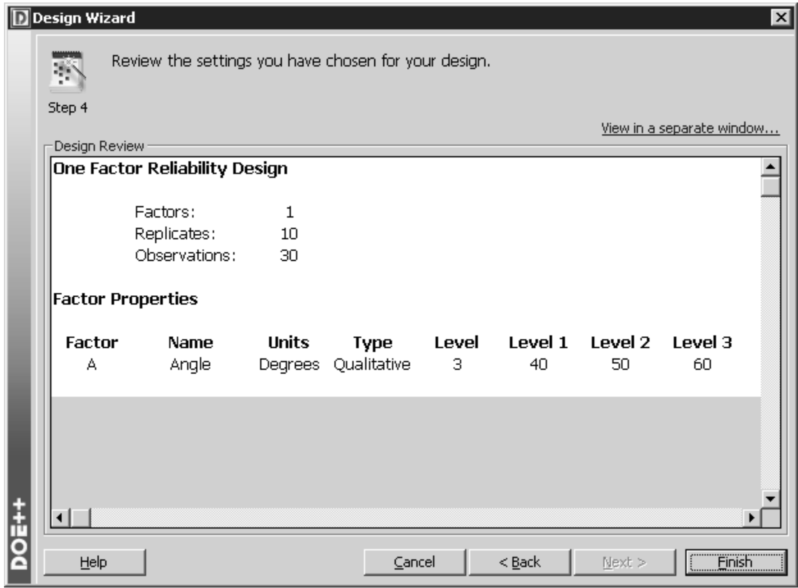

- Click the **Factor Properties** button. In the Factor Properties window that appears, enter the name and units for the factor then specify the number of levels and enter the level values, as shown next.




Note that the factor in a one factor reliability design is always qualitative.

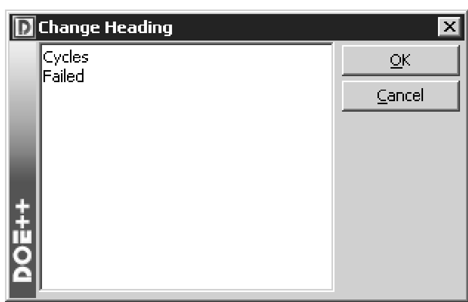
- Click **OK** to close the Factor Properties window and return to the Design Wizard.
- Click **Next** to proceed to the next step.

- The fourth step of the Design Wizard provides a review of the settings you have chosen for the design.

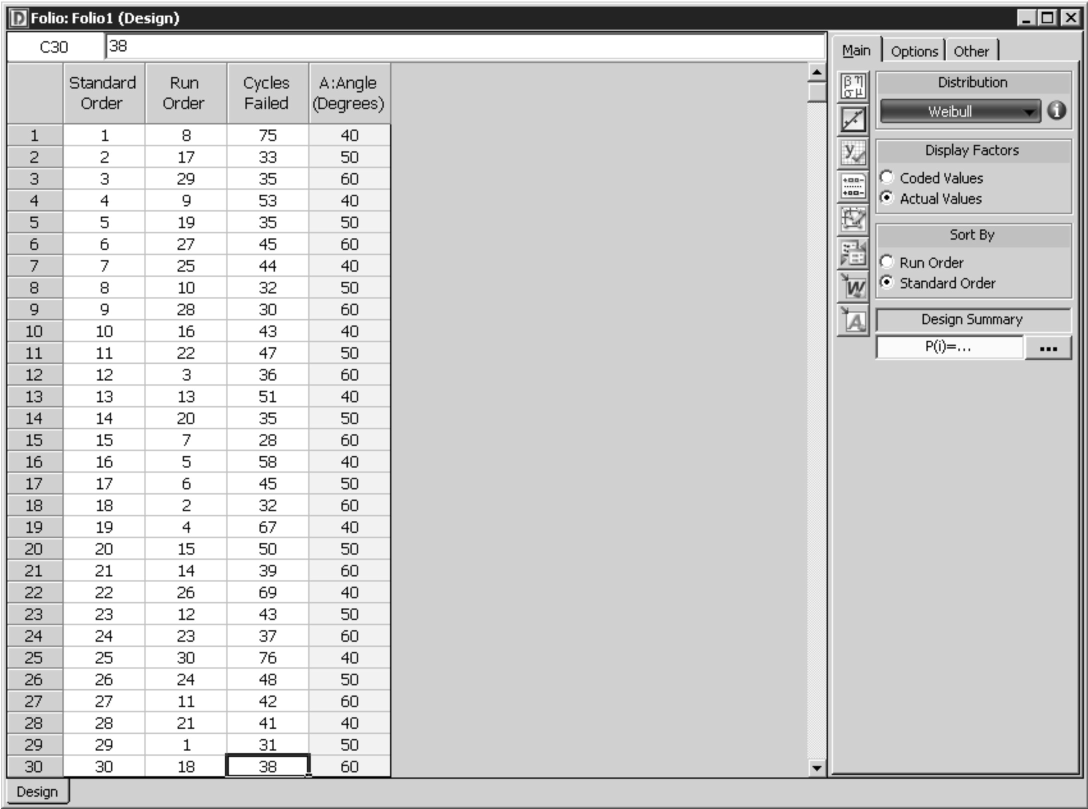



If any of the settings are not as intended, you can click the **Back** button to return to the appropriate step and make any necessary changes.

- Click **Finish** to create the new project with the Standard Folio containing the one factor reliability design.
- Save the project as "OneFactorReliability.rso7."
- To change the title of the results column (*i.e.* "Time Failed"), double-click the column header and, in the window that appears, type **Cycles Failed**, as shown next, then click **OK**.



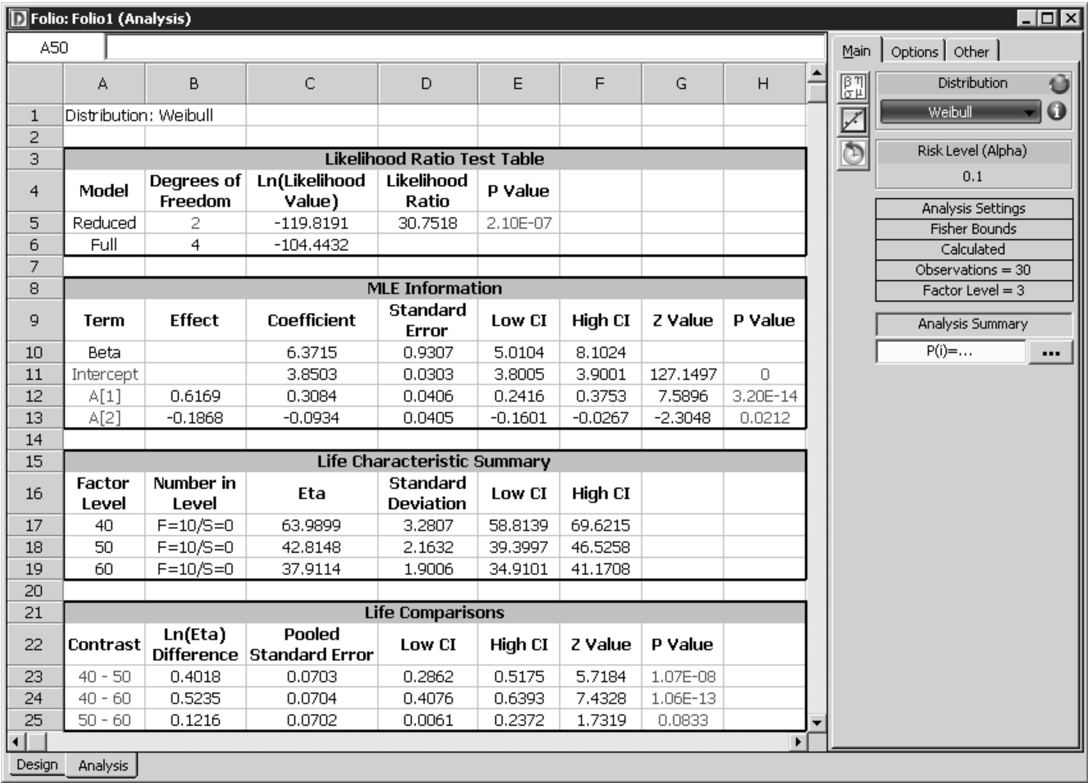
- Select **Standard Order** in the **Sort By** area in the Control Panel to sort the Data Sheet by the Standard Order column, then enter the response data in the order given in this example on page 93. The Folio will look like the one shown next.




| | Standard Order | Run Order | Cycles Failed | A:Angle (Degrees) |
|---|---|---|---|---|
| 1 | 1 | 8 | 75 | 40 |
| 2 | 2 | 17 | 33 | 50 |
| 3 | 3 | 29 | 35 | 60 |
| 4 | 4 | 9 | 53 | 40 |
| 5 | 5 | 19 | 35 | 50 |
| 6 | 6 | 27 | 45 | 60 |
| 7 | 7 | 25 | 44 | 40 |
| 8 | 8 | 10 | 32 | 50 |
| 9 | 9 | 28 | 30 | 60 |
| 10 | 10 | 16 | 43 | 40 |
| 11 | 11 | 22 | 47 | 50 |
| 12 | 12 | 3 | 36 | 60 |
| 13 | 13 | 13 | 51 | 40 |
| 14 | 14 | 20 | 35 | 50 |
| 15 | 15 | 7 | 28 | 60 |
| 16 | 16 | 5 | 58 | 40 |
| 17 | 17 | 6 | 45 | 50 |
| 18 | 18 | 2 | 32 | 60 |
| 19 | 19 | 4 | 67 | 40 |
| 20 | 20 | 15 | 50 | 50 |
| 21 | 21 | 14 | 39 | 60 |
| 22 | 22 | 26 | 69 | 40 |
| 23 | 23 | 12 | 43 | 50 |
| 24 | 24 | 23 | 37 | 60 |
| 25 | 25 | 30 | 76 | 40 |
| 26 | 26 | 24 | 48 | 50 |
| 27 | 27 | 11 | 42 | 60 |
| 28 | 28 | 21 | 41 | 40 |
| 29 | 29 | 1 | 31 | 50 |
| 30 | 30 | 18 | 38 | 60 |

- On the Main page of the Control Panel, select the **Weibull** distribution.
- On the Options page of the Control Panel, specify a **Risk Level (Alpha)** value of **0.1**.

- Return to the Main page of the Control Panel and click **Calculate**. The Analysis tab is shown next.




Distribution: Weibull

| Likelihood Ratio Test Table | | | | |
|---|---|---|---|---|
| Model | Degrees of Freedom | Ln(Likelihood Value) | Likelihood Ratio | P Value |
| Reduced | 2 | -119.8191 | 30.7518 | 2.10E-07 |
| Full | 4 | -104.4432 | | |

| MLE Information | | | | | | | |
|---|---|---|---|---|---|---|---|
| Term | Effect | Coefficient | Standard Error | Low CI | High CI | Z Value | P Value |
| Beta | | 6.3715 | 0.9307 | 5.0104 | 8.1024 | | |
| Intercept | | 3.8503 | 0.0303 | 3.8005 | 3.9001 | 127.1497 | 0 |
| A[1] | 0.6169 | 0.3084 | 0.0406 | 0.2416 | 0.3753 | 7.5896 | 3.20E-14 |
| A[2] | -0.1868 | -0.0934 | 0.0405 | -0.1601 | -0.0267 | -2.3048 | 0.0212 |

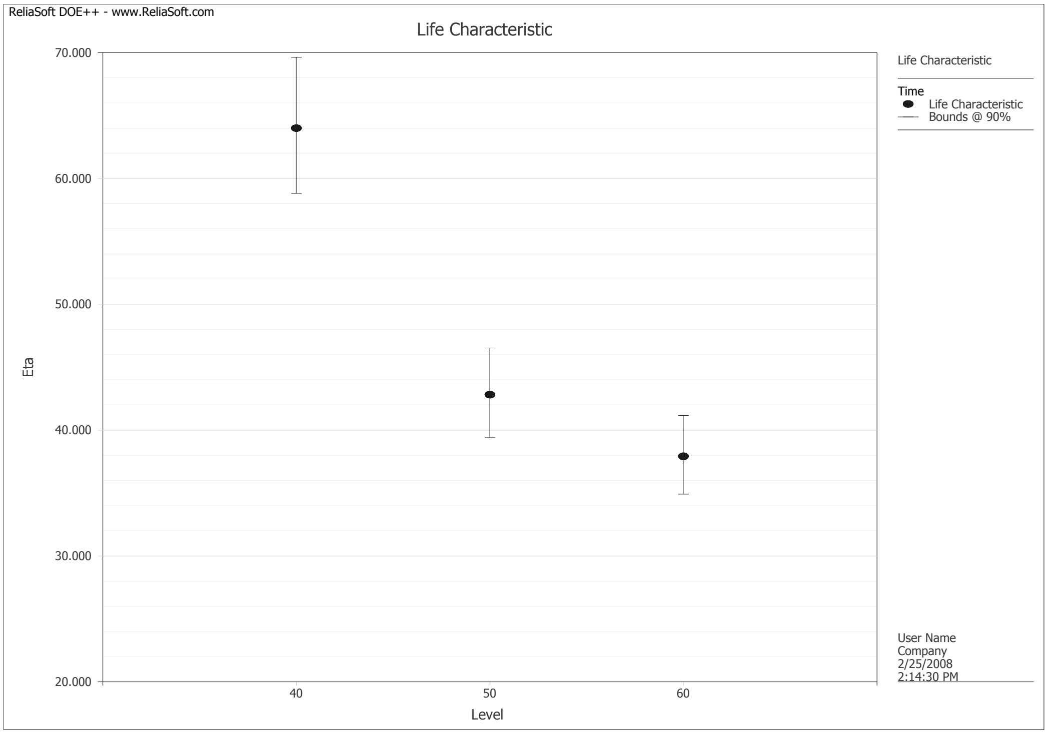
| Life Characteristic Summary | | | | | |
|---|---|---|---|---|---|
| Factor Level | Number in Level | Eta | Standard Deviation | Low CI | High CI |
| 40 | F=10/S=0 | 63.9899 | 3.2807 | 58.8139 | 69.6215 |
| 50 | F=10/S=0 | 42.8148 | 2.1632 | 39.3997 | 46.5258 |
| 60 | F=10/S=0 | 37.9114 | 1.9006 | 34.9101 | 41.1708 |

| Life Comparisons | | | | | | |
|---|---|---|---|---|---|---|
| Contrast | Ln(Eta) Difference | Pooled Standard Error | Low CI | High CI | Z Value | P Value |
| 40 - 50 | 0.4018 | 0.0703 | 0.2862 | 0.5175 | 5.7184 | 1.07E-08 |
| 40 - 60 | 0.5235 | 0.0704 | 0.4076 | 0.6393 | 7.4328 | 1.06E-13 |
| 50 - 60 | 0.1216 | 0.0702 | 0.0061 | 0.2372 | 1.7319 | 0.0833 |

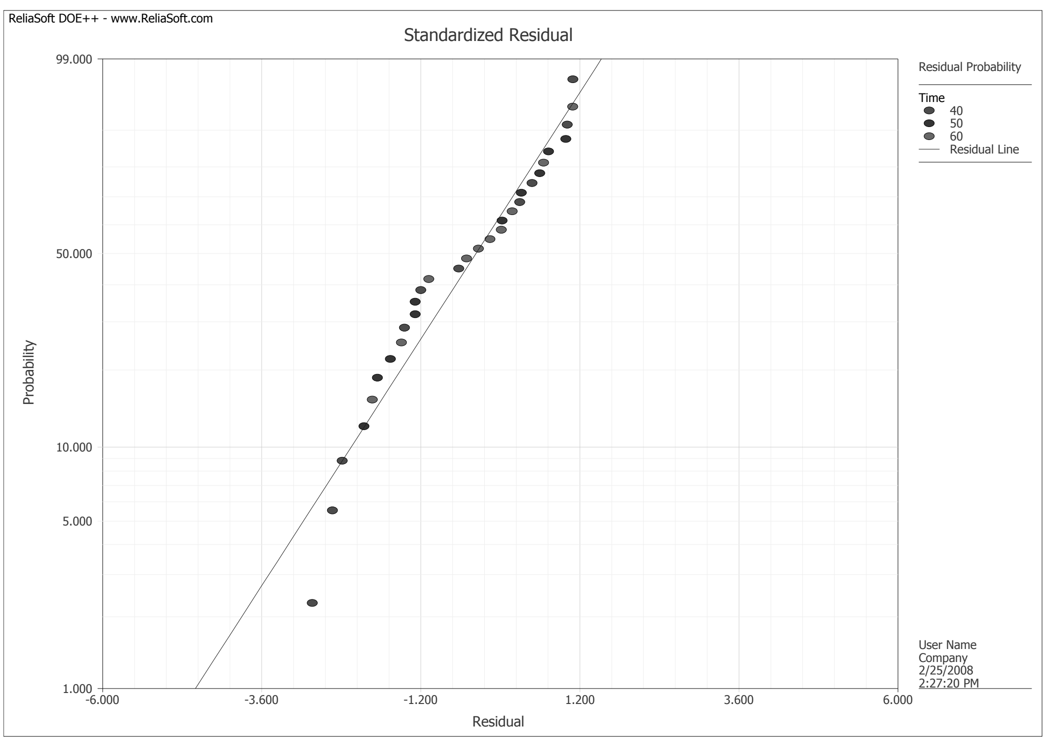
The *p* values in the Life Comparisons table indicate that, at a significance level of 10%, bending the units at 50 and 60 degrees has a significant effect on the life of the units such that the units are expected to fail faster than they would being bent at 40 degrees.

- To confirm this graphically, click the **Plot** icon to add the Plot tab to the Folio.

- Select **Life Characteristic** from the **Plot Type** menu in the Control Panel. The Life Characteristic plot is shown next.




- To validate the normality assumption, select **Residual Probability** from the **Plot Type** menu in the Control Panel. The Residual Probability plot is shown next.




The data points follow the line reasonably closely, indicating that the normality assumption is valid.

- Save and close the project then proceed to the next example.

## 4.10 Example 10: Two Level Fractional Factorial Reliability Design

A reliability engineer wishes to predict the reliability of an electromechanical component in the field. In order to build a life-stress relationship for reliability prediction, important stresses need to be identified. Given the large number of possible factors, the engineer chooses a $2^{8-4}$ fractional factorial design (*i.e.* 16 runs) with the following factors:

A: Temperature
B: Humidity
C: Load
D: Speed
E: Voltage
F: Coolant Type
G: Vibration
H: Current

Since the ultimate goal is to build a reliability model, a reliability design is chosen. Units are inspected every 20 hours and the test is suspended after 200 hours of testing. The data set is shown next.

| Temperature | Humidity | Load | Speed | Voltage | Coolant Type | Vibration | Current | Last Inspected | State (F or S) | Time to F or S |
|---|---|---|---|---|---|---|---|---|---|---|
| -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 180 | F | 200 |
| 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | 60 | F | 80 |
| -1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | 140 | F | 160 |
| 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | 180 | F | 200 |
| -1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | 160 | F | 180 |
| 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 80 | F | 100 |
| -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 180 | S | 200 |
| 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 80 | F | 100 |
| -1 | -1 | -1 | 1 | -1 | 1 | 1 | 1 | 180 | S | 200 |
| 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 180 | S | 200 |
| -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 120 | S | 140 |
| 1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | 120 | F | 140 |
| -1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 | 180 | S | 200 |
| 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 120 | F | 140 |
| -1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 140 | F | 160 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 120 | F | 140 |

Note that the factor values in this data set are expressed as coded values, where -1 represents the factor's low value and 1 represents the factor's high value.

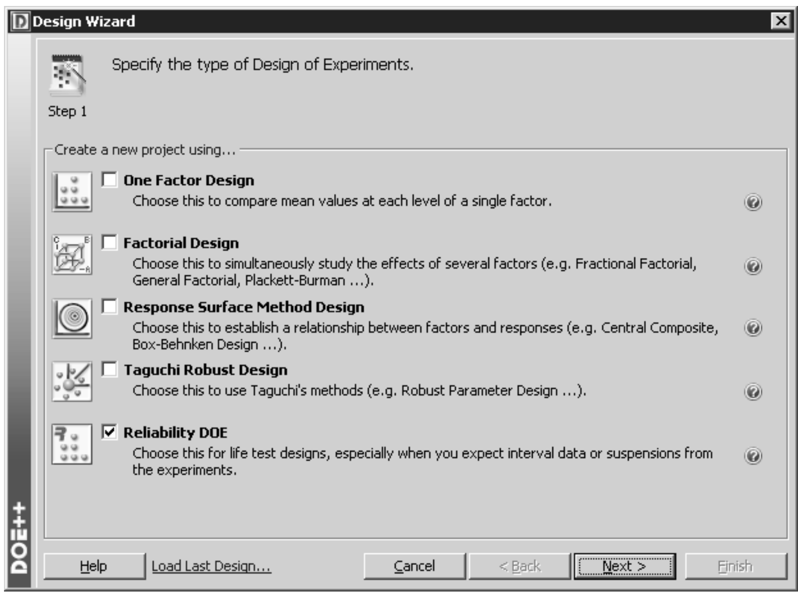
Do the following:

- Identify the significant factor(s).

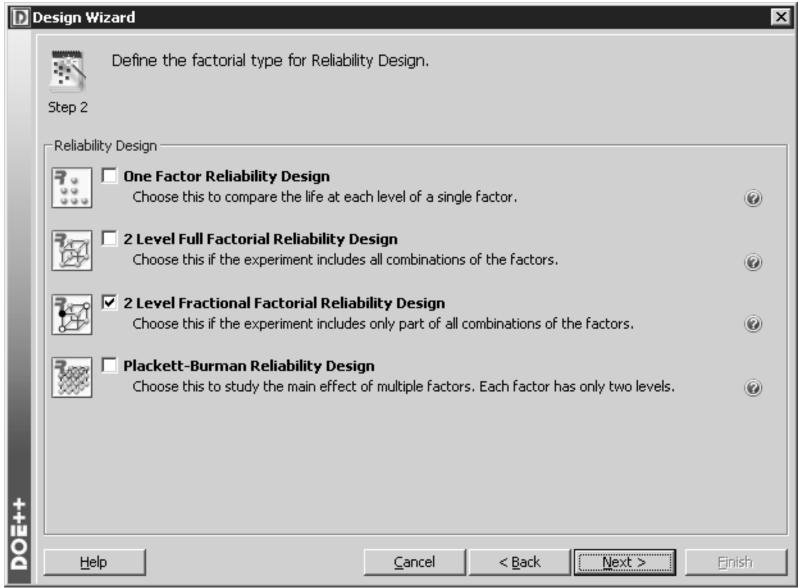
*The sample file for this example is located in the "Training Guide" folder in your application directory (e.g. C:\Program Files\ReliaSoft\DOE\Training Guide) and is named "TwoLevelFractionalReliability.rdoe." The data set for this example is available in the Spreadsheet in the "TwoLevelFractionalReliability-Data.rso7" file located in the "Raw Data" folder within the "Training Guide" folder.*

**Solution**

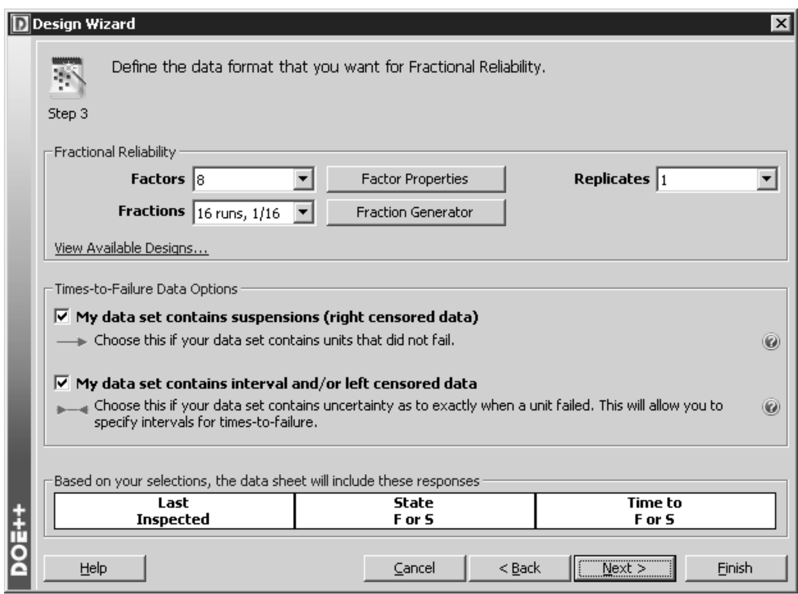
- Create a new project. On the first page of the Design Wizard, select to create a **Reliability DOE** and click **Next** to proceed to the next step.



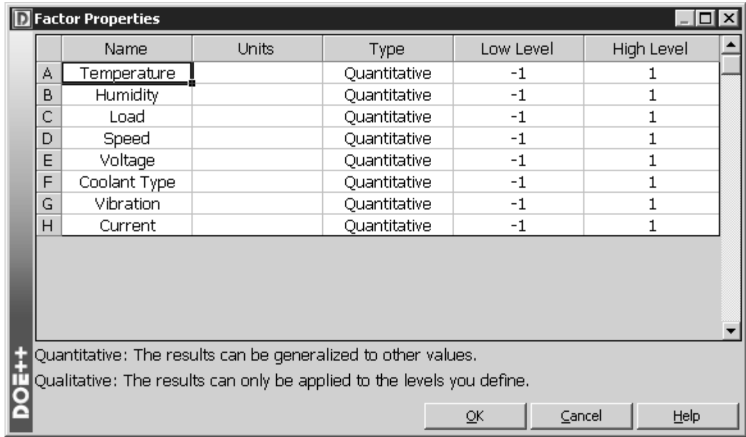
- In the second step of the Design Wizard, select **2 Level Fractional Factorial Reliability Design** and click **Next** to proceed to the next step.




- In the third step of the Design Wizard, select **8** from the **Factors** drop-down and **16 runs, 1/16** from the **Fractions** drop-down. In the Times-to-Failure Data Options area, select both **My data set contains suspensions (right censored data)** and **My data set contains interval and/or left censored data**, as shown next.

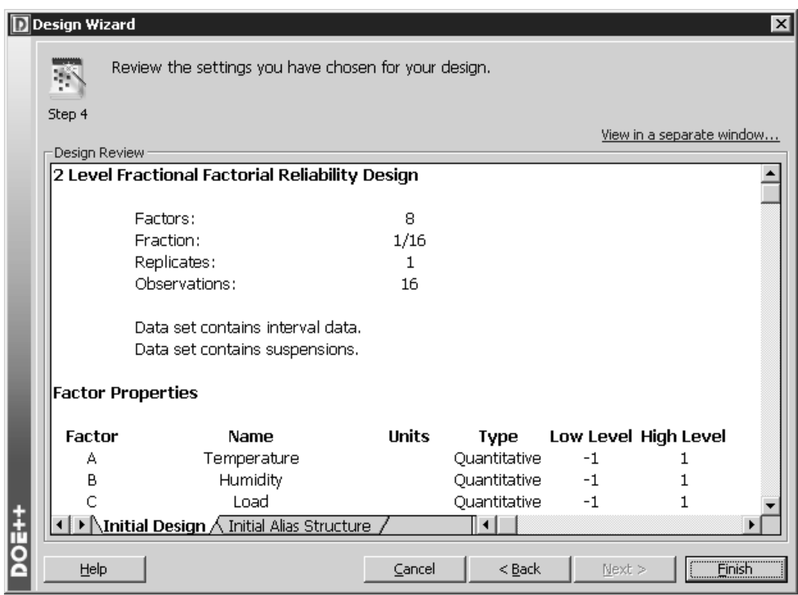


- Click the **Factor Properties** button. In the Factor Properties window that appears, enter the names for the factors, as shown next.

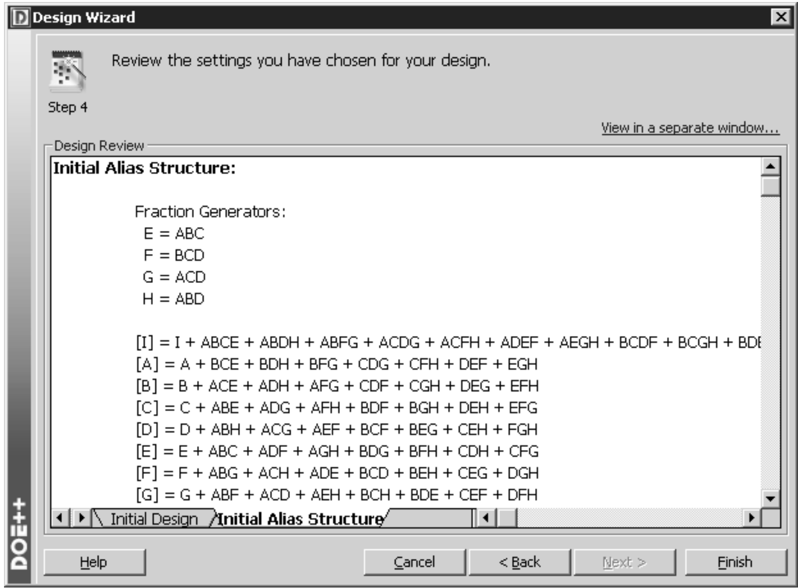



Because only coded values are used in this example, you do not need to enter low and high values for the factors.

- Click **OK** to close the Factor Properties window and return to the Design Wizard.
- For this example, we will use the default fraction generators, so there is no need to access the Fraction Generator window. Click **Next** to proceed to the next step.
- The fourth step of the Design Wizard provides a review of the settings you have chosen for the design.

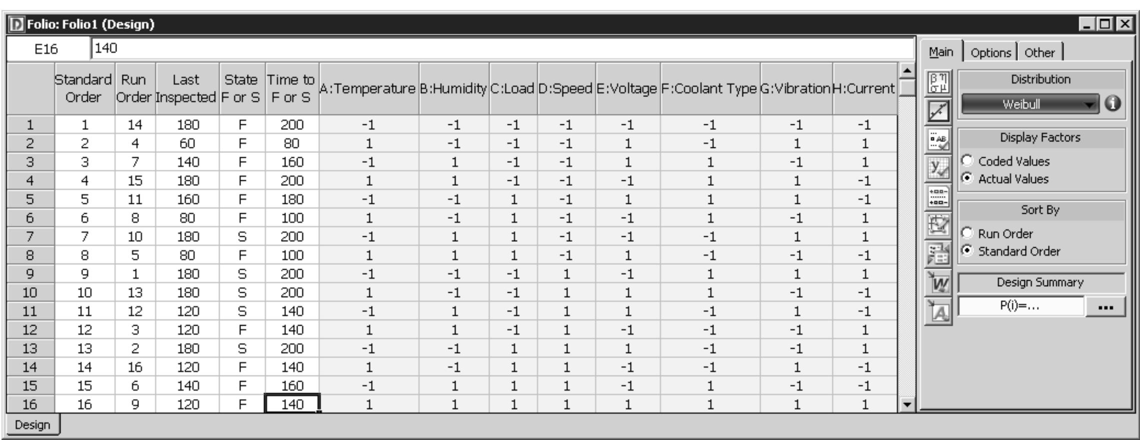


Note that there is an additional tab called “Initial Alias Structure” in the summary area. Click this tab to view the alias structure of the design, as shown next.




If any of the settings are not as intended, you can click the **Back** button to return to the appropriate step and make any necessary changes.

- Click **Finish** to create the new project with the Standard Folio containing the two level fractional factorial reliability design.
- Save the project as “TwoLevelFractionalReliability.rso7.”
- Select **Standard Order** in the **Sort By** area in the Control Panel to sort the Data Sheet by the Standard Order column, then enter the response data in the order given in this example on page 101. The Folio will look like the one shown next.



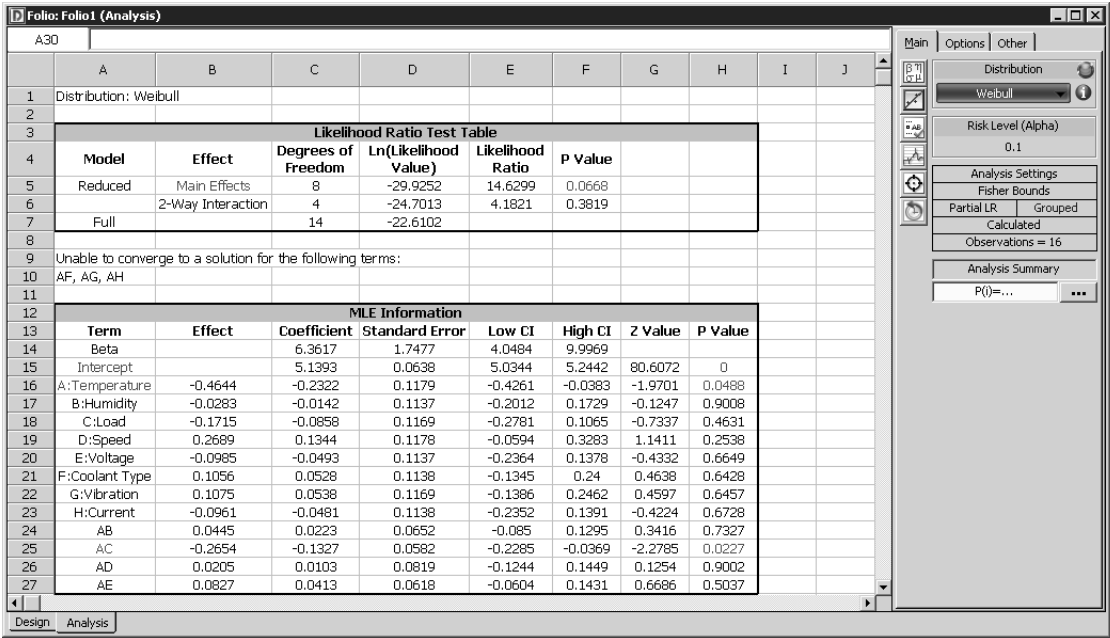
| | Standard Order | Run Order | Last Inspected | State F or S | Time to F or S | A:Temperature | B:Humidity | C:Load | D:Speed | E:Voltage | F:Coolant Type | G:Vibration | H:Current |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 14 | 180 | F | 200 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 2 | 2 | 4 | 60 | F | 80 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 |
| 3 | 3 | 7 | 140 | F | 160 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 |
| 4 | 4 | 15 | 180 | F | 200 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 |
| 5 | 5 | 11 | 160 | F | 180 | -1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 |
| 6 | 6 | 8 | 80 | F | 100 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 |
| 7 | 7 | 10 | 180 | S | 200 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 |
| 8 | 8 | 5 | 80 | F | 100 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 |
| 9 | 9 | 1 | 180 | S | 200 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | 1 |
| 10 | 10 | 13 | 180 | S | 200 | 1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 |
| 11 | 11 | 12 | 120 | S | 140 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 |
| 12 | 12 | 3 | 120 | F | 140 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 |
| 13 | 13 | 2 | 180 | S | 200 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 |
| 14 | 14 | 16 | 120 | F | 140 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 |
| 15 | 15 | 6 | 140 | F | 160 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 |
| 16 | 16 | 9 | 120 | F | 140 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |



Because you did not change the factor values from the original coded values in the Factor Properties window, the Folio will look the same regardless of whether **Coded Values** or **Actual Values** is selected in the **Display Factors** area of the Control Panel.

- On the Main page of the Control Panel, select the **Weibull** distribution.
- On the Options page of the Control Panel, specify a **Risk Level (Alpha)** value of **0.1** and select to use **Grouped Terms**.

- Return to the Main page of the Control Panel and click **Calculate**. The Analysis tab is shown next.



Distribution: Weibull

| Model | Effect | Degrees of Freedom | Ln(Likelihood Value) | Likelihood Ratio | P Value |
|---|---|---|---|---|---|
| Reduced | Main Effects | 8 | -29.9252 | 14.6299 | 0.0668 |
| | 2-Way Interaction | 4 | -24.7013 | 4.1821 | 0.3819 |
| Full | | 14 | -22.6102 | | |

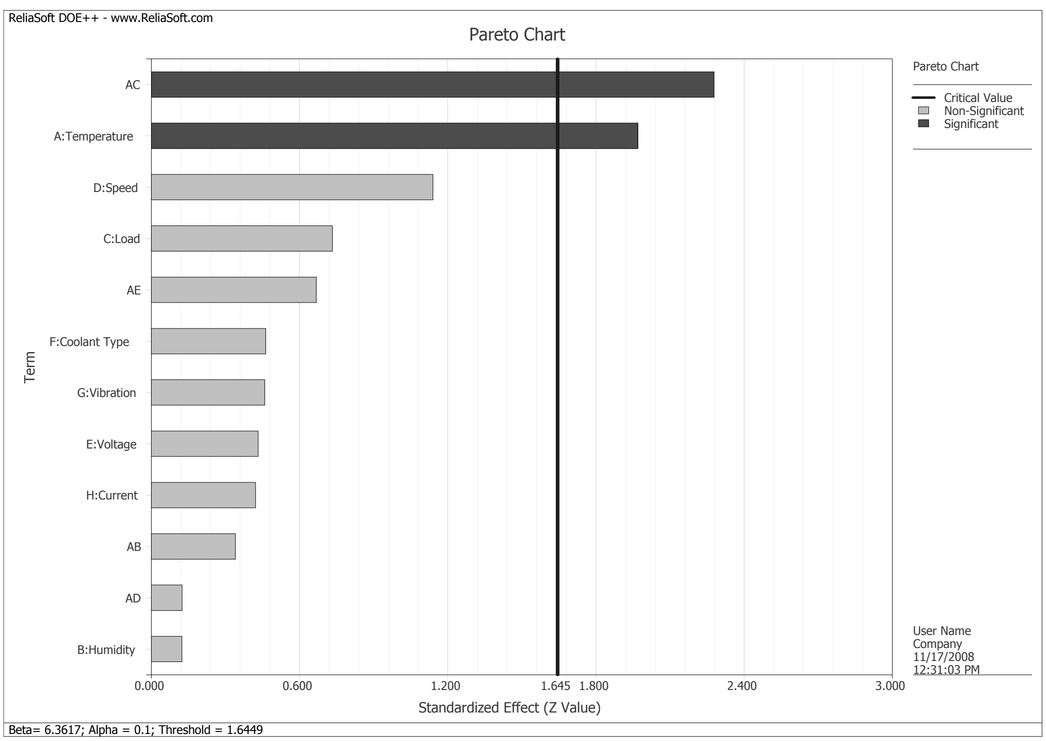
Unable to converge to a solution for the following terms:
AF, AG, AH

| Term | Effect | Coefficient | Standard Error | Low CI | High CI | Z Value | P Value |
|---|---|---|---|---|---|---|---|
| Beta | | 6.3617 | 1.7477 | 4.0484 | 9.9969 | | |
| Intercept | | 5.1393 | 0.0638 | 5.0344 | 5.2442 | 80.6072 | 0 |
| A:Temperature | -0.4644 | -0.2322 | 0.1179 | -0.4261 | -0.0383 | -1.9701 | 0.0488 |
| B:Humidity | -0.0283 | -0.0142 | 0.1137 | -0.2012 | 0.1729 | -0.1247 | 0.9008 |
| C:Load | -0.1715 | -0.0858 | 0.1169 | -0.2781 | 0.1065 | -0.7337 | 0.4631 |
| D:Speed | 0.2689 | 0.1344 | 0.1178 | -0.0594 | 0.3283 | 1.1411 | 0.2538 |
| E:Voltage | -0.0985 | -0.0493 | 0.1137 | -0.2364 | 0.1378 | -0.4332 | 0.6649 |
| F:Coolant Type | 0.1056 | 0.0528 | 0.1138 | -0.1345 | 0.24 | 0.4638 | 0.6428 |
| G:Vibration | 0.1075 | 0.0538 | 0.1169 | -0.1386 | 0.2462 | 0.4597 | 0.6457 |
| H:Current | -0.0961 | -0.0481 | 0.1138 | -0.2352 | 0.1391 | -0.4224 | 0.6728 |
| AB | 0.0445 | 0.0223 | 0.0652 | -0.085 | 0.1295 | 0.3416 | 0.7327 |
| AC | -0.2654 | -0.1327 | 0.0582 | -0.2285 | -0.0369 | -2.2785 | 0.0227 |
| AD | 0.0205 | 0.0103 | 0.0819 | -0.1244 | 0.1449 | 0.1254 | 0.9002 |
| AE | 0.0827 | 0.0413 | 0.0618 | -0.0604 | 0.1431 | 0.6686 | 0.5037 |



The *p* values in the MLE Information table indicate that temperature and the interaction between temperature and load (AC) are significant factors at the 10% significance level.

- To confirm this graphically, click the **Plot** icon to add the Plot tab to the Folio.
- Select **Pareto Chart** from the **Plot Type** menu in the Control Panel. The Pareto chart is shown next.



Temperature and the interaction between temperature and load are confirmed as the significant factors at the 10% significance level. The engineer can now proceed to building the life-stress relationship with the important factors.

- Save and close the project then proceed to the next example.

## 4.11 Example 11: Multiple Linear Regression Tool

In doing some research, an investor decides to explore the relationship between the stock price of Apple Inc. (AAPL) and its iPod sales.

The data set is shown next.[8]

| Date | AAPL Stock Price ($) | iPods Sold (M) |
|---|---|---|
| Jan-02 | 10.13 | 125 |
| Apr-02 | 11.68 | 57 |
| Jul-02 | 10.88 | 54 |
| Oct-02 | 7.42 | 140 |
| Jan-03 | 7.65 | 219 |
| Apr-03 | 7.25 | 78 |
| Jul-03 | 8.54 | 304 |
| Aug-03 | 10.74 | 336 |
| Jan-04 | 10.86 | 733 |
| Apr-04 | 12.25 | 807 |
| Jul-04 | 14.4 | 860 |
| Oct-04 | 17.6 | 2016 |
| Jan-05 | 30.64 | 4580 |
| Apr-05 | 41.66 | 5311 |
| Jul-05 | 37.54 | 6155 |
| Oct-05 | 47.72 | 6451 |
| Jan-06 | 65.77 | 14043 |
| Apr-06 | 68.91 | 8526 |
| Jul-06 | 62.48 | 8111 |
| Oct-06 | 70.93 | 8729 |
| Jan-07 | 85.86 | 21066 |
| Apr-07 | 87.75 | 10549 |
| Jul-07 | 114.34 | 9815 |
| Oct-07 | 141.24 | 10200 |
| Jan-08 | 190.08 | 22121 |

8. In this data set, each stock price is the average of the monthly adjusted close price for the preceding three months, taken from http://finance.yahoo.com/q/hp?s=AAPL&a=09&b=1&c=2001&d=11&e=31&f=2007&g=m&z=66&y=66. Quarterly sales reports are taken from the U.S. Securities and Exchange Commission Web site at http://www.sec.gov/cgi-bin/browse-edgar?type=10-q&dateb=&owner=include&count=40&action=getcompany&CIK=AAPL.

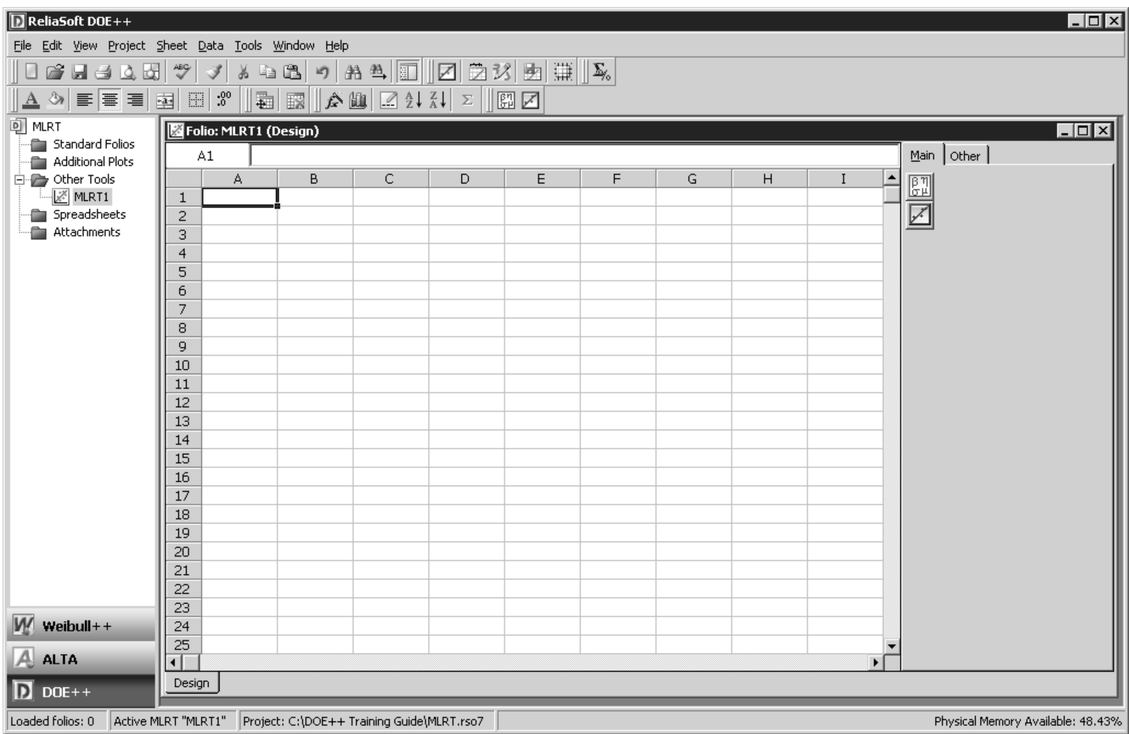
Do the following:

- Determine if a relationship exists between stock price and sales, then build a model.
- Determine if a transformation might be appropriate and build a new model if desirable.

*The sample file for this example is located in the "Training Guide" folder in your application directory (e.g. C:\Program Files\ReliaSoft\DOE\Training Guide) and is named "MLRT.rdoe." The data set for this example is available in the Spreadsheet in the "MLRT-Data.rso7" file located in the "Raw Data" folder within the "Training Guide" folder.*

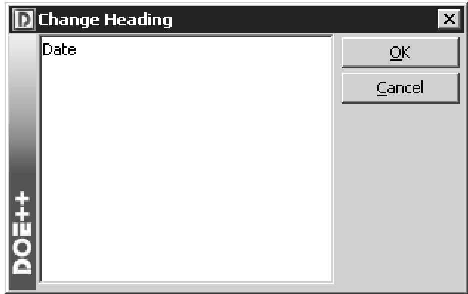
**Solution**

- Create a new project. On the first page of the Design Wizard, clear all checkboxes and click **Finish** to create an empty project.
- Save the project as "MLRT.rso7."
- Select **Add Other Tools** then **Add Multiple Linear Regression** from the **Project** menu. A blank Multiple Linear Regression tool will be added to the project, as shown next.

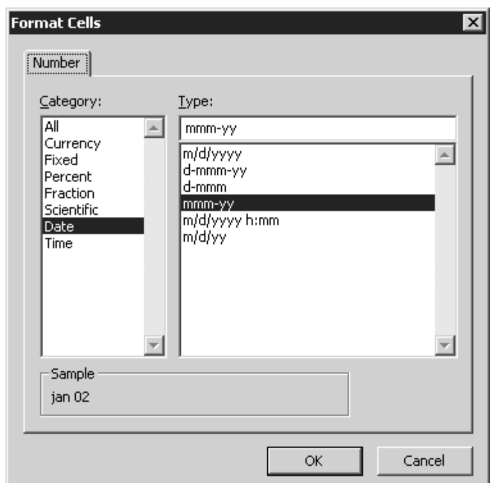



The Multiple Linear Regression Tool (MLRT) allows you to enter your own experimental data without using one of the design types provided in the software. You can then analyze the data in much the same way that designed experimental data sets are analyzed in the Standard Folio.

- Double-click the column header of column A and, in the window that appears, type **Date**, as shown next, then click **OK**.

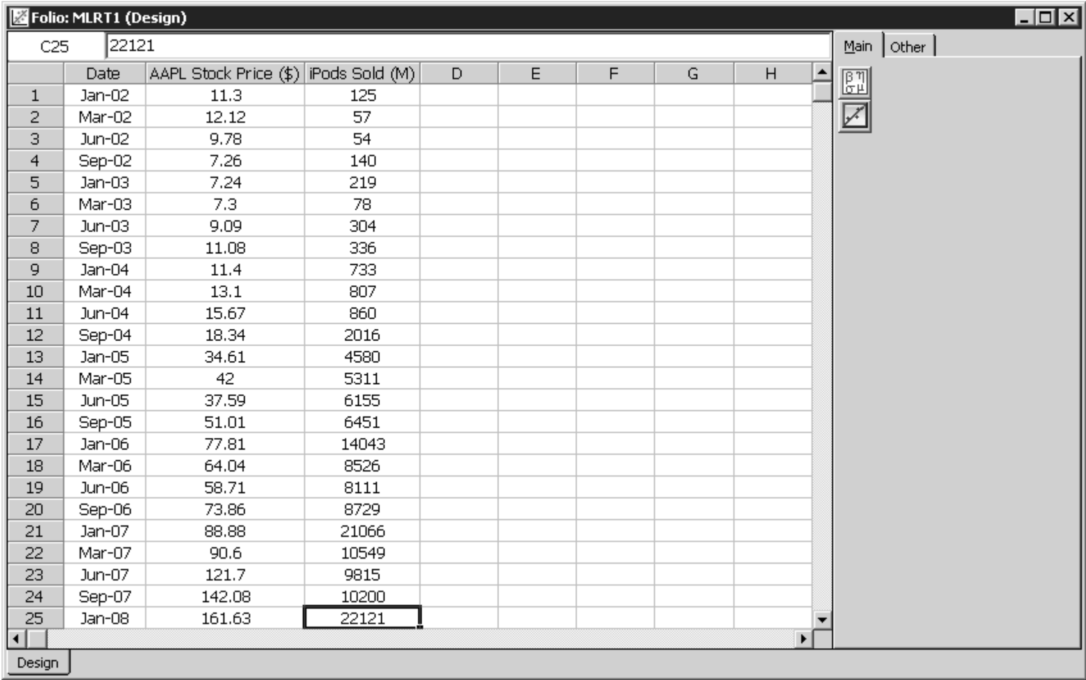



- Using the same process, change column B's title to **AAPL Stock Price ($)** and column C's title to **iPods Sold (M)**.
- Right-click the column header of the Date column and select **Format Selection** then **Custom Number** from the shortcut menu.
- In the Format Cells window that appears, select **Date** in the Category area and then **mmm-yy** in the Type area, as shown next.



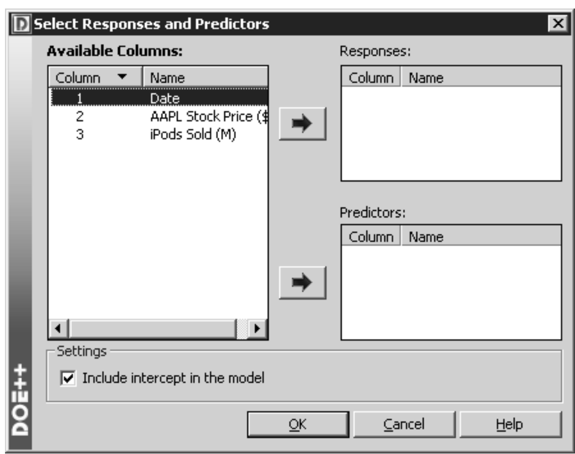

- Click **OK** to accept your selections and close the window.

- Enter the data given on page 107. For each date in the Date column, enter the date as the first of the month (*e.g.* for Jan-02, type **1/1/02**). The MLRT will look like the one shown next.




| | Date | AAPL Stock Price ($) | iPods Sold (M) | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | Jan-02 | 11.3 | 125 | | | | | |
| 2 | Mar-02 | 12.12 | 57 | | | | | |
| 3 | Jun-02 | 9.78 | 54 | | | | | |
| 4 | Sep-02 | 7.26 | 140 | | | | | |
| 5 | Jan-03 | 7.24 | 219 | | | | | |
| 6 | Mar-03 | 7.3 | 78 | | | | | |
| 7 | Jun-03 | 9.09 | 304 | | | | | |
| 8 | Sep-03 | 11.08 | 336 | | | | | |
| 9 | Jan-04 | 11.4 | 733 | | | | | |
| 10 | Mar-04 | 13.1 | 807 | | | | | |
| 11 | Jun-04 | 15.67 | 860 | | | | | |
| 12 | Sep-04 | 18.34 | 2016 | | | | | |
| 13 | Jan-05 | 34.61 | 4580 | | | | | |
| 14 | Mar-05 | 42 | 5311 | | | | | |
| 15 | Jun-05 | 37.59 | 6155 | | | | | |
| 16 | Sep-05 | 51.01 | 6451 | | | | | |
| 17 | Jan-06 | 77.81 | 14043 | | | | | |
| 18 | Mar-06 | 64.04 | 8526 | | | | | |
| 19 | Jun-06 | 58.71 | 8111 | | | | | |
| 20 | Sep-06 | 73.86 | 8729 | | | | | |
| 21 | Jan-07 | 88.88 | 21066 | | | | | |
| 22 | Mar-07 | 90.6 | 10549 | | | | | |
| 23 | Jun-07 | 121.7 | 9815 | | | | | |
| 24 | Sep-07 | 142.08 | 10200 | | | | | |
| 25 | Jan-08 | 161.63 | 22121 | | | | | |

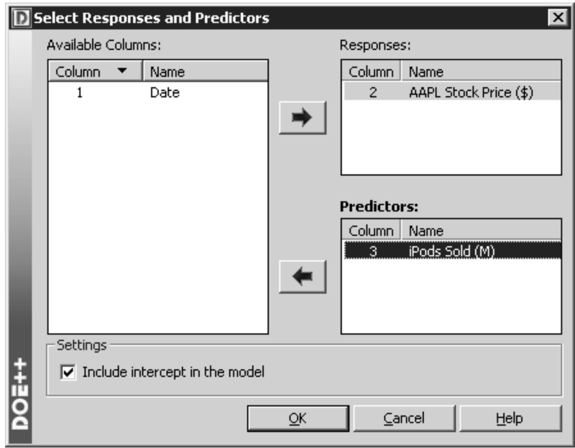
- Click **Calculate** to analyze the data set. The Select Responses and Predictors window will open, as shown next.



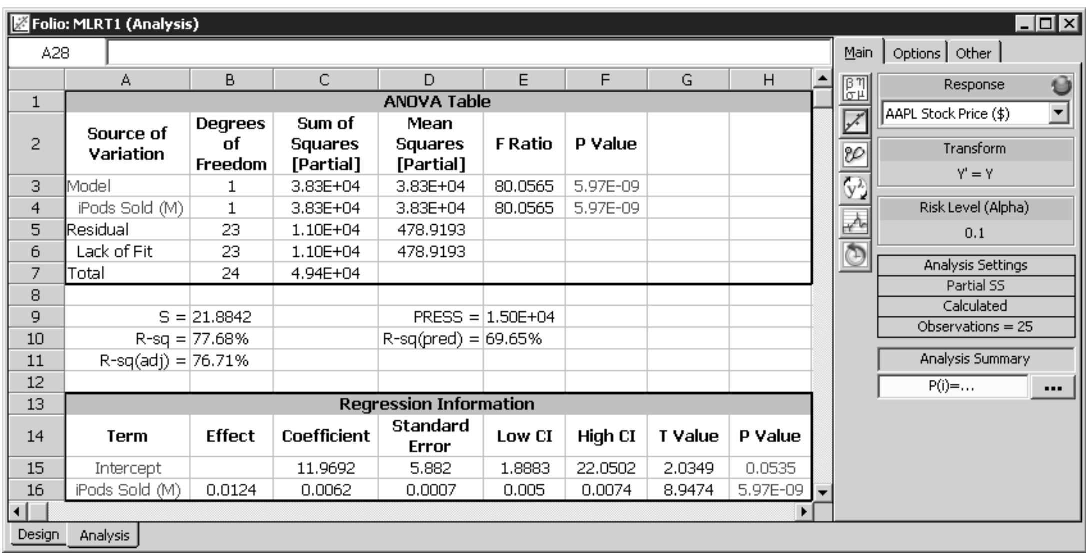

The Select Responses and Predictors window allows you to specify for each column of data you have entered in the Design tab of the MLRT whether it represents factor settings or response data.

- Select column 2, **AAPL Stock Price ($)**, in the **Available Columns** area then click the arrow beside the **Responses** area to move the selected column to the **Responses** area, indicating that the data contained in that column are response data.

- Select column 3, **iPods Sold (M)**, in the **Available Columns** area then click the arrow beside the **Predictors** area to move the selected column to the **Predictors** area, indicating that the data contained in that column are factor settings. The window will look like the one shown next.




- Click **OK** to close the Select Responses and Predictors window and continue with the analysis. The Analysis tab will be added to the MLRT, as shown next.




| ANOVA Table | | | | | |
|---|---|---|---|---|---|
| Source of Variation | Degrees of Freedom | Sum of Squares [Partial] | Mean Squares [Partial] | F Ratio | P Value |
| Model | 1 | 3.83E+04 | 3.83E+04 | 80.0565 | 5.97E-09 |
| iPods Sold (M) | 1 | 3.83E+04 | 3.83E+04 | 80.0565 | 5.97E-09 |
| Residual | 23 | 1.10E+04 | 478.9193 | | |
| Lack of Fit | 23 | 1.10E+04 | 478.9193 | | |
| Total | 24 | 4.94E+04 | | | |

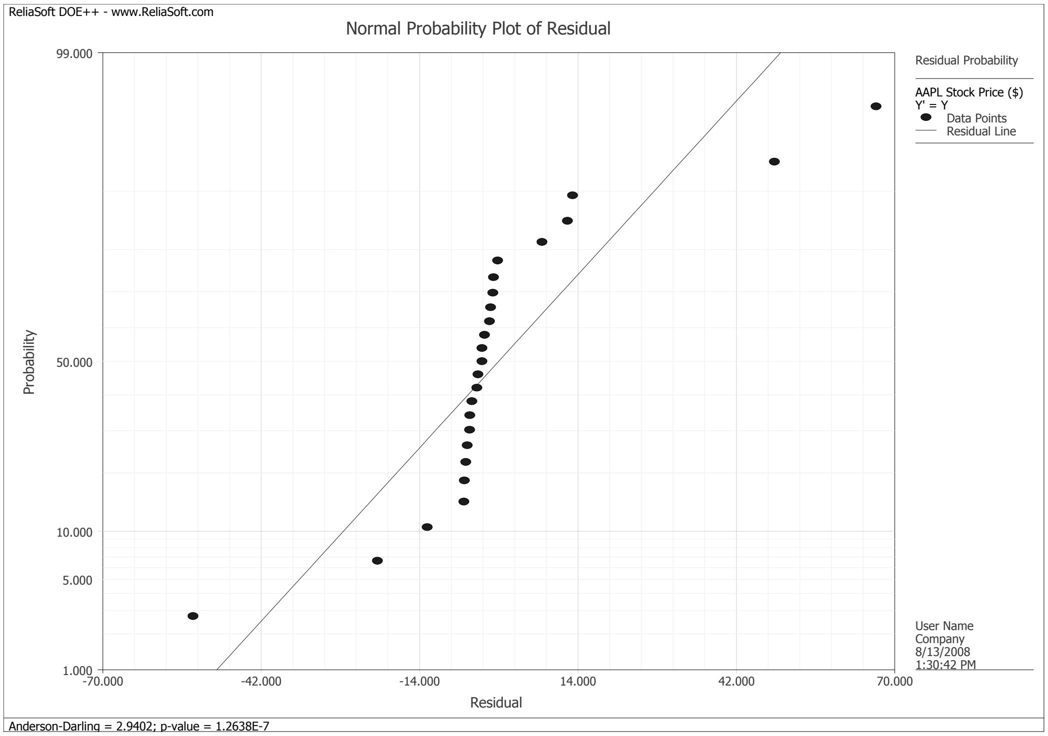
| | | | |
|---|---|---|---|
| S = | 21.8842 | PRESS = | 1.50E+04 |
| R-sq = | 77.68% | R-sq(pred) = | 69.65% |
| R-sq(adj) = | 76.71% | | |

| Regression Information | | | | | | | |
|---|---|---|---|---|---|---|---|
| Term | Effect | Coefficient | Standard Error | Low CI | High CI | T Value | P Value |
| Intercept | | 11.9692 | 5.882 | 1.8883 | 22.0502 | 2.0349 | 0.0535 |
| iPods Sold (M) | 0.0124 | 0.0062 | 0.0007 | 0.005 | 0.0074 | 8.9474 | 5.97E-09 |

The *p* values in the ANOVA and Regression Information tables indicate that the number of iPods sold is a significant factor in the change of the stock price.

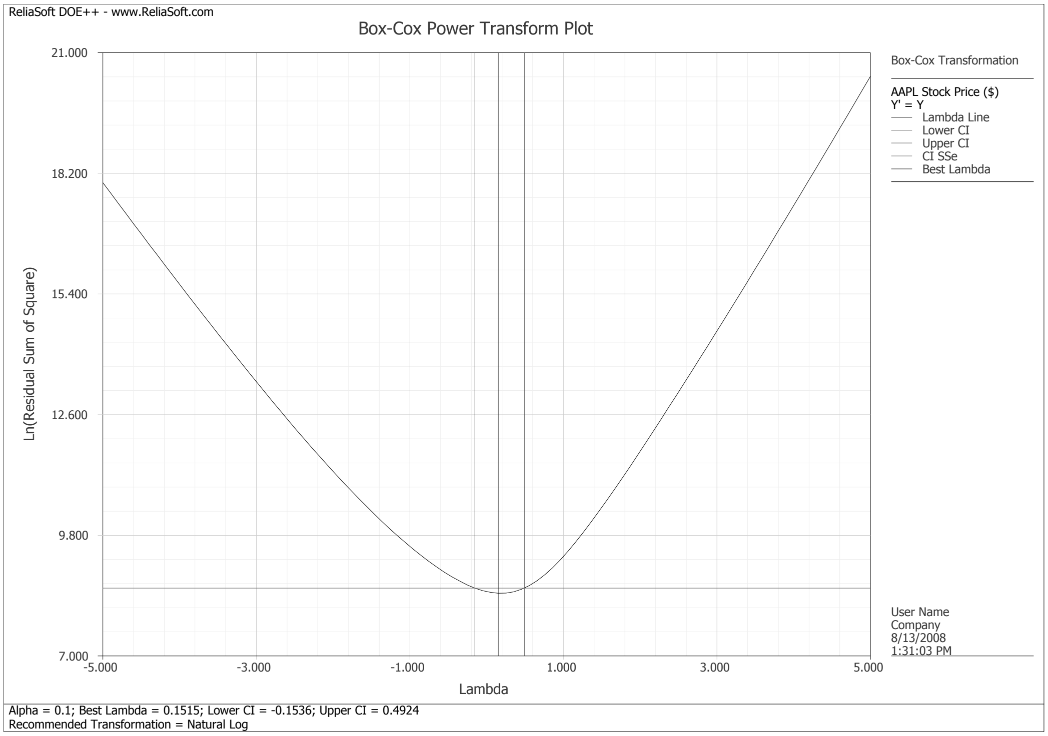
- To determine if the model is acceptable or if a transformation would be appropriate, click the **Plot** icon to add the Plot tab to the MLRT.

- Select **Residual Probability** from the **Plot Type** menu in the Control Panel. The Residual Probability plot is shown next.




You will recall that if the model accurately accounts for the effect of the factor, the residuals (*i.e.* the differences between the observed response values at each factor level and the response values predicted by the model) should be approximately normally and independently distributed. The noticeable pattern in the way the data points deviate from the residual line here suggest that Y is not normally distributed and that a transformation may be appropriate. The Box-Cox Transformation plot can help in determining the transformation that should be used.

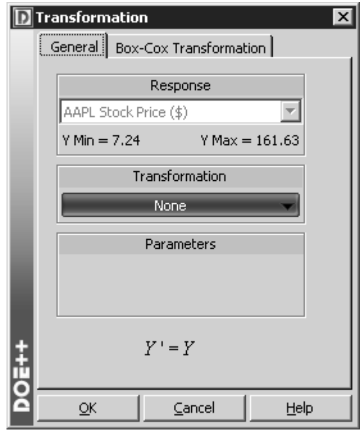
- Select **Box-Cox Transformation** from the **Plot Type** menu in the Control Panel. The Box-Cox Transformation plot is shown next.



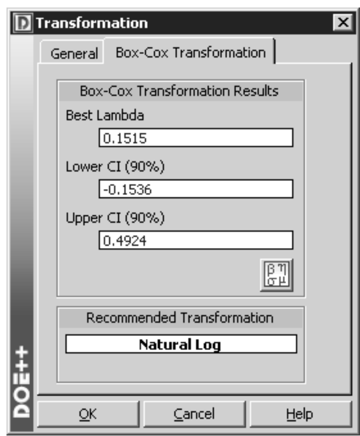

Based on the best (lowest) value of lambda, the recommended transformation is the natural log transformation, as displayed in the lower title of the plot.

- Return to the Analysis tab by clicking its page index tab.
- Click the **Transformation** icon.

- The Transformation window will open, as shown next.[9]



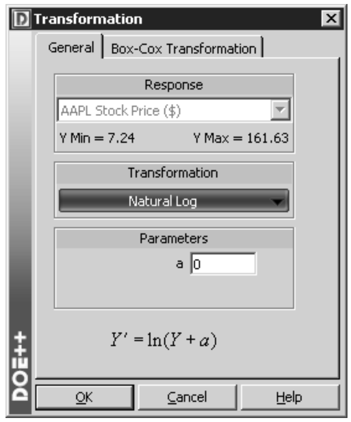

- Click the **Box-Cox Transformation** tab at the top of the window to go to the Box-Cox Transformation page. This page can help to select a transformation by giving you the same results as the Box-Cox Transformation plot. Click **Calculate** to calculate the best lambda value along with its confidence interval. The recommended transformation, based on the best lambda value, is displayed at the bottom of the window, as shown next.



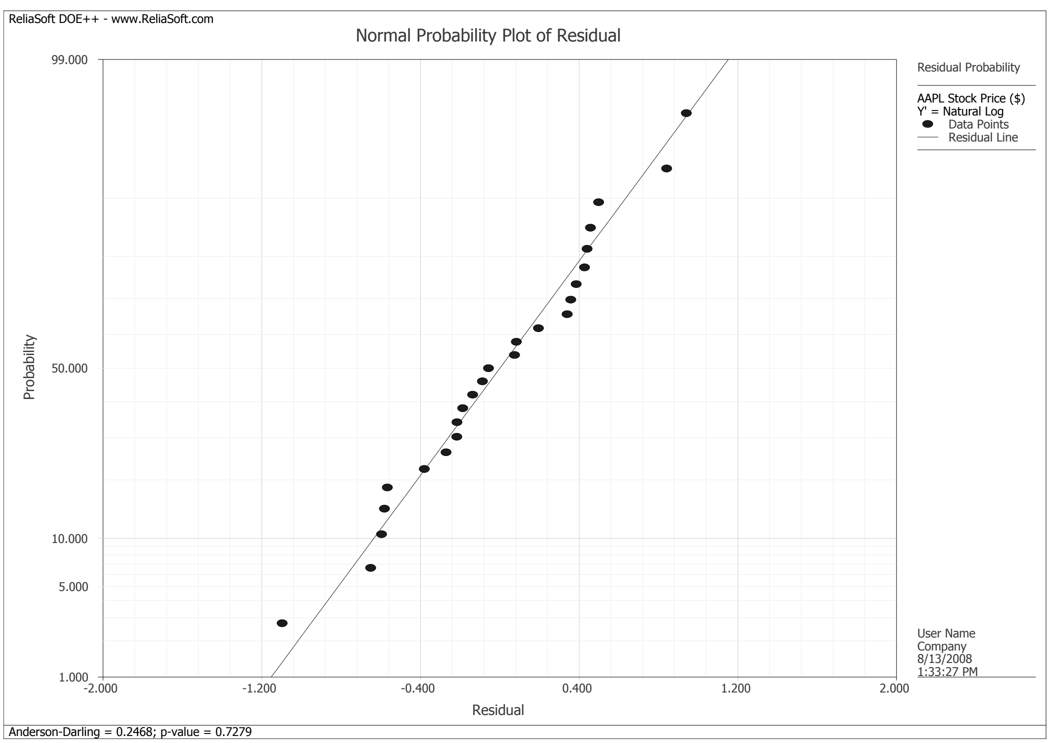

You can see that the recommended transformation is the same as the one recommended on the Box-Cox Transformation plot.

9. You will notice that the **Response** drop-down is unavailable. This is because the window was accessed from the Analysis tab Control Panel, so the choice you make will be applied only to the response currently selected in the Control Panel. When you access the Transformation window from the Standard Folio Design tab Control Panel, the **Response** drop-down is enabled.

- Return to the General page of the Transformation window and select **Natural Log** from the **Transformation** menu. The equation for the transformation is now displayed at the bottom of the window. Any recommended parameters are displayed in the **Parameters** area, as shown next.




- Click **OK** to apply the transformation to the response and close the window. You will see that the status indicator in the Response area of the Control Panel has turned red, indicating that changes have been made and the results in the Analysis tab are no longer valid.
- Click the **Plot** icon to recalculate the data set with the new transformation applied and display the results in a plot.
- Select **Residual Probability** from the **Plot Type** menu in the Control Panel. The new Residual Probability plot is shown next.




The data points now follow the residual line reasonably closely, indicating that the transformation was, in fact, appropriate.

- Save and close the project.

# Practice Questions 5

This section contains some additional practice questions and the next section presents solutions to these questions. The file for these examples is located in the "Training Guide" folder in your application directory (*e.g.* C:\Program Files\ReliaSoft\DOE\Training Guide) and is named "Practice Questions.rdoe."

## 5.1 Practice Question 1

In comparing post-surgery rehabilitation programs after knee reconstruction, a researcher divides patients into 4 different groups (A through D). After 6 months of recovery, the costs incurred by each group are compared. Costs include expenses such as professional physical therapy, medications, post-surgery doctor visits, etc. Note that the recovery results target was attained for all patients.

| Group A Costs ($) | Group B Costs ($) | Group C Costs ($) | Group D Costs ($) |
|---|---|---|---|
| 360 | 770 | 570 | 330 |
| 460 | 930 | 730 | 440 |
| 540 | 1070 | 870 | 550 |
| 590 | 1160 | 960 | 620 |
| 480 | 960 | 760 | 470 |
| 500 | 990 | 790 | 500 |
| 340 | 850 | 850 | 290 |
| 470 | 940 | 740 | 460 |
| 570 | 1110 | 910 | 580 |
| 480 | 970 | 770 | 480 |

1. Create a new project and Standard Folio containing a one factor design. Name the Folio "Question1."
2. Enter the data.
3. Validate the normality assumption using the Residual Probability plot.
4. Determine whether the four programs are significantly different in terms of cost.
5. Using the Box plot, determine which program is the least expensive.
6. Save the project as "Practice Questions.rso7." Close the Folio but leave the project open.

## 5.2 Practice Question 2

A metal deposition process consists of depositing a thin film of metal over steel sheets. The deposition thickness needs to meet a target of 2mm. Minimum variability is desired in the thickness across the metal sheet. Several factors are thought to affect this process, namely the speed at which the sheet moves through the deposition chamber, the pressure in the chamber, the temperature to which the metals that will cover the sheet are heated and the order in which these metals are laid. Due to the high number of out-of-spec sheets, the experimenter decides to use a Taguchi robust design to find the appropriate control factor settings to make the process more robust.

The following are the settings for the control factors:

| Control Factor | Level 1 | Level 2 | Level 3 |
|---|---|---|---|
| Speed | Low | Medium | High |
| Temperature | Low | Medium | High |
| Pressure | Low | Medium | High |
| Order | Arrangement 1 | Arrangement 2 | Arrangement 3 |

The following deposition thickness data, in mm, are collected at 5 different predetermined locations across the metal sheet:

| Control Factor Settings | | | | Location 1 | Location 2 | Location 3 | Location 4 | Location 5 |
|---|---|---|---|---|---|---|---|---|
| Speed | Temperature | Pressure | Order of Metals | | | | | |
| Low | Low | Low | Arrangement 1 | 2.007 | 2.038 | 2.04 | 2.051 | 2.025 |
| Low | Medium | Medium | Arrangement 2 | 1.983 | 1.972 | 2.057 | 2.033 | 2.043 |
| Low | High | High | Arrangement 3 | 1.979 | 2 | 2.002 | 2.055 | 2.033 |
| Medium | Low | Medium | Arrangement 3 | 1.995 | 1.996 | 2.01 | 2.025 | 2.101 |
| Medium | Medium | High | Arrangement 1 | 1.984 | 1.996 | 1.996 | 2.028 | 2.031 |
| Medium | High | Low | Arrangement 2 | 1.995 | 1.987 | 1.99 | 2.01 | 2.015 |
| High | Low | High | Arrangement 2 | 1.97 | 1.99 | 1.994 | 2.009 | 2.013 |
| High | Medium | Low | Arrangement 3 | 1.976 | 1.965 | 1.983 | 1.988 | 2.033 |
| High | High | Medium | Arrangement 1 | 1.907 | 1.938 | 1.974 | 1.99 | 2.005 |

1. Create a new Standard Folio for a Taguchi robust design. For the inner array, use a Taguchi L9(3^4) orthogonal array. For the outer array, use a general full factorial design with 1 factor and 5 levels. Name the Folio “Question2.”
2. Analyze the data using a Nominal signal-to-noise ratio.
3. Determine the optimum settings for the control factors based on the Main Effect plots.
4. Save your file and close the Folio.

## 5.3 Practice Question 3

A design engineer would like to optimize the design of a capacitor such that the life will be maximized. The three factors that will be included in the study are the height and length of the capacitor and the type of solder used. He chooses to use a full factorial reliability design and use two replicates. Factors are set at the limits of the allowed range for this application. Due to the test conditions and resources, units can only be inspected every 24 hours. The data set is shown next.

| Solder Type | Height (microns) | Length (microns) | Last Inspected | State (F or S) | Time to F or S |
|---|---|---|---|---|---|
| S1 | 50 | 200 | 168 | F | 192 |
| S2 | 50 | 200 | 168 | F | 192 |
| S1 | 100 | 200 | 264 | F | 288 |
| S2 | 100 | 200 | 216 | F | 240 |
| S1 | 50 | 600 | 216 | S | 240 |
| S2 | 50 | 600 | 168 | F | 192 |
| S1 | 100 | 600 | 216 | F | 240 |
| S2 | 100 | 600 | 192 | F | 216 |
| S1 | 50 | 200 | 192 | F | 216 |
| S2 | 50 | 200 | 216 | S | 240 |
| S1 | 100 | 200 | 288 | F | 312 |
| S2 | 100 | 200 | 312 | F | 336 |
| S1 | 50 | 600 | 216 | F | 240 |
| S2 | 50 | 600 | 216 | F | 240 |
| S1 | 100 | 600 | 264 | F | 288 |
| S2 | 100 | 600 | 312 | F | 336 |

Note that two units were taken out of the experiment for further study after 240 hours due to anomalies observed in the units.

1. Create a new Standard Folio for a two level full factorial reliability design with right and interval censored data. Name the Folio "Question3."
2. Analyze the data using the Weibull distribution.
3. Validate the model assumptions using the Residual Probability plot.
4. Identify the important effects.
5. Reduce the model accordingly and recalculate.
6. What would be the optimal factor settings for this design?
7. Save your file and close the Folio.

## 5.4 Practice Question 4

The design of an automobile fuel cone is thought to be affected by six factors in the manufacturing process:

- Factor A: cavity temperature
- Factor B: core temperature
- Factor C: melt temperature
- Factor D: hold pressure
- Factor E: injection speed

- Factor F: cool time

The manufacturer of the fuel cone is unable to run the $2^6 = 64$ runs required to complete one replicate for a two level full factorial experiment with six factors. Instead, they decide to run a fractional factorial design. Considering that three-factor and higher-order interactions are likely to be inactive, the manufacturer selects a $2^{6-2}$ (resolution IV) design that will require only 16 runs. The data set is shown next.

| Cavity Temperature | Core Temperature | Melt Temperature | Hold Pressure | Injection Speed | Cool Time | Response |
|---|---|---|---|---|---|---|
| -1 | -1 | -1 | -1 | -1 | -1 | 7 |
| 1 | -1 | -1 | -1 | 1 | -1 | 7 |
| -1 | 1 | -1 | -1 | 1 | 1 | 6.5 |
| 1 | 1 | -1 | -1 | -1 | 1 | 7 |
| -1 | -1 | 1 | -1 | 1 | 1 | 5 |
| 1 | -1 | 1 | -1 | -1 | 1 | 5 |
| -1 | 1 | 1 | -1 | -1 | -1 | 3.5 |
| 1 | 1 | 1 | -1 | 1 | -1 | 4.5 |
| -1 | -1 | -1 | 1 | -1 | 1 | 7 |
| 1 | -1 | -1 | 1 | 1 | 1 | 7.5 |
| -1 | 1 | -1 | 1 | 1 | -1 | 7 |
| 1 | 1 | -1 | 1 | -1 | -1 | 4.5 |
| -1 | -1 | 1 | 1 | 1 | -1 | 14 |
| 1 | -1 | 1 | 1 | -1 | -1 | 11 |
| -1 | 1 | 1 | 1 | -1 | 1 | 13 |
| 1 | 1 | 1 | 1 | 1 | 1 | 13.5 |

1. Create a new Standard Folio for a two level fractional factorial design using coded values. Name the Folio “Question4.”
2. Examine the fraction generators and the alias structure. State the assumptions of this design and what can be done to mitigate problems.
3. Identify the significant effects using the Effect Probability plot and reduce the model accordingly.
4. Save your file and close the Folio.

## 5.5 Practice Question 5

In “Example 7: Sequential Optimization” on page 70 of Chapter 4, we assumed that the experimenter was concerned only about the yield of the chemical reaction and selected optimal settings of temperature and reaction time based on that requirement. Often, this is not the case; instead, it might be necessary to optimize a number of responses simultaneously. Assume that the cost of making the product and the pH of the product are responses that have to be accounted for when selecting optimum settings. While the cost is to be minimized, the pH should be within a close range to 7. The following data are collected:

| Center Point | Temperature (F) | Reaction Time (min) | Yield (%) | Cost ($) | pH |
|---|---|---|---|---|---|
| 1 | 345 | 155 | 89.75 | 420.5 | 7.98 |
| 1 | 355 | 155 | 90.2 | 419.6 | 7.56 |
| 1 | 345 | 175 | 92 | 416 | 7.14 |
| 1 | 355 | 175 | 94.25 | 411.5 | 6.08 |
| 1 | 342.9289322 | 165 | 90.5 | 419 | 7.46 |
| 1 | 357.0710678 | 165 | 92.75 | 414.5 | 6.5 |
| 1 | 350 | 150.8578644 | 88.4 | 423.2 | 6.92 |
| 1 | 350 | 179.1421356 | 92.6 | 414.8 | 6.24 |
| 0 | 350 | 165 | 94.85 | 410.3 | 7.16 |
| 0 | 350 | 165 | 95.45 | 409.1 | 6.6 |
| 0 | 350 | 165 | 95 | 410 | 7.02 |
| 0 | 350 | 165 | 94.55 | 410.9 | 6.78 |
| 0 | 350 | 165 | 94.7 | 410.6 | 7.2 |

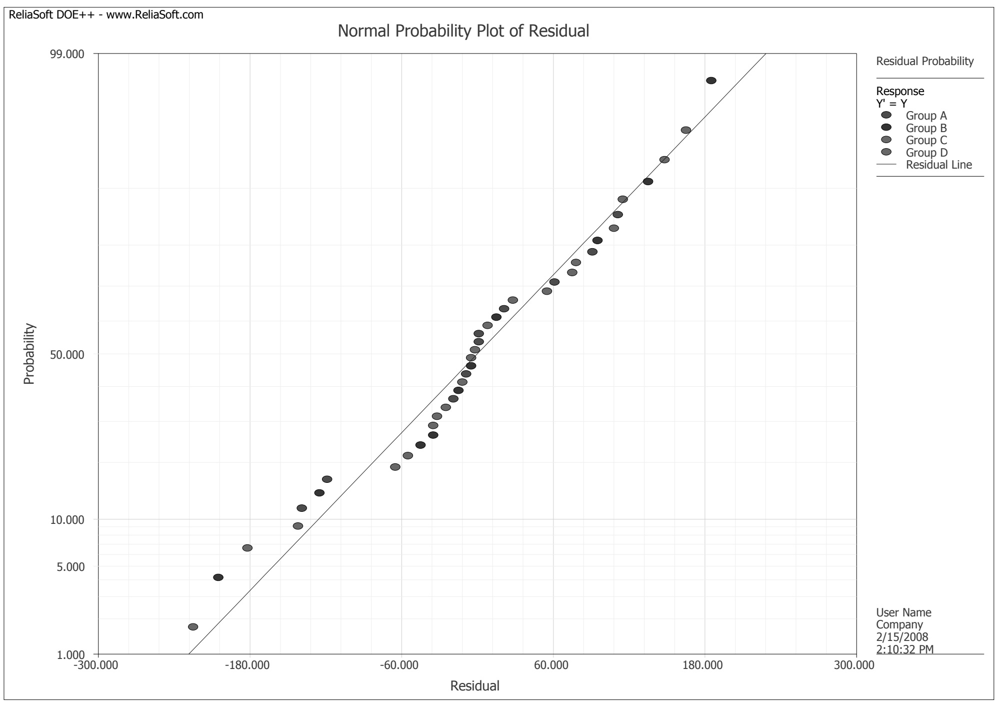
1. Create a new Standard Folio for a central composite design with three responses: yield (%), cost ($) and pH. Use five center points and $\alpha = 1.4142$. Use the factor ranges from the central composite design in the original example (*i.e.* 345-355$^{o}$F for temperature and 155-175 minutes for reaction time). Name the Folio "Question5."
2. Reduce the model for each response accordingly.
3. Identify optimal factor settings such that the yield is maximized, the cost is minimized and the pH stays on target at 7. Assume that the experimenter wants to have a target yield value of 95 although any value of yield greater than 94 is acceptable. For the cost, assume that the experimenter wants to lower the cost to 400 although any cost value below 415 is acceptable. For the pH, a target of 7 is desired but values between 6.9 and 7.1 are also acceptable. Assume equal weights and importance for all responses.

# Answers to Practice Questions 6

This section provides answers to the Practice Questions in Chapter 5.

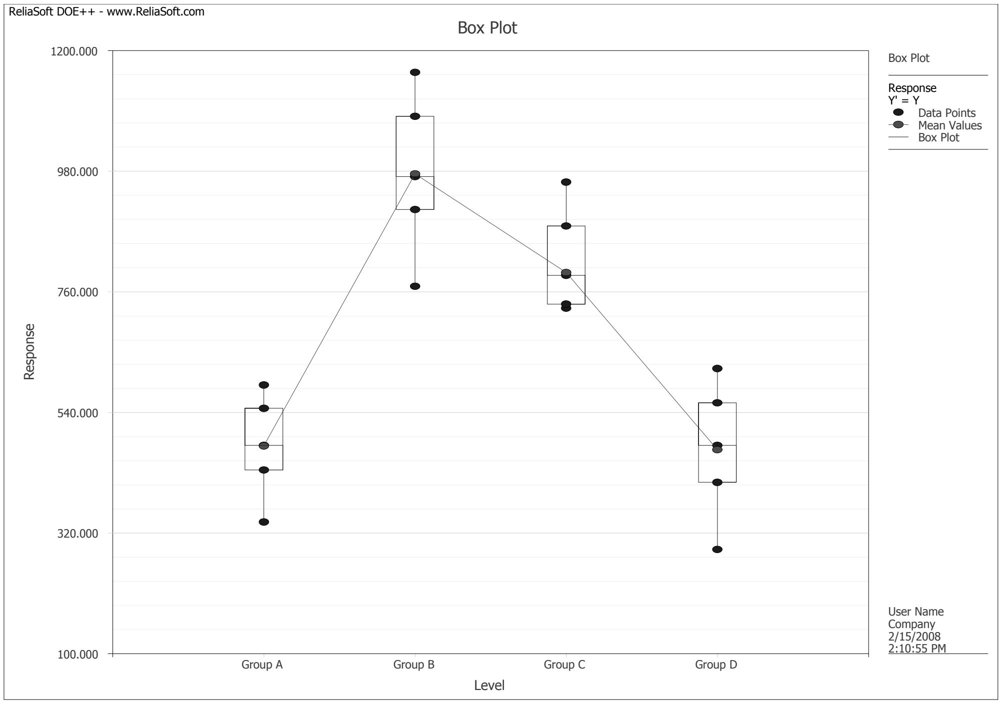
## 6.1 Practice Question 1

3. There is no apparent deviation from the normal probability line.




4. From the Mean Comparisons table, there is significant difference between all programs except between programs A and D.

5. From the Box plot, programs A and D are least expensive.




## 6.2 Practice Question 2

4. By examining the Main Effect plots for Y Mean and Signal Noise Ratio, you can see that the medium setting for the speed is the best choice for maximizing the signal-to-noise ratio while still remaining on target. The Main Effect plot for Y Std indicates that the medium setting also has the smallest standard deviation value.

   For the temperature, the low and high levels are slightly better than the medium level for maximizing the signal-to-noise ratio and minimizing standard deviation; however, the medium setting results in values that are closest to the 2mm thickness target. The medium level is chosen.

   The pressure setting does not seem to considerably affect the deposition thickness. The largest signal-to-noise ratio and the smallest Y Std are found at the low setting; therefore, it is preferable.

   The order of the metals does not seem to considerably affect the deposition thickness. The largest signal-to-noise ratio and the smallest standard deviation values are observed at the second arrangement setting, which is therefore preferable.

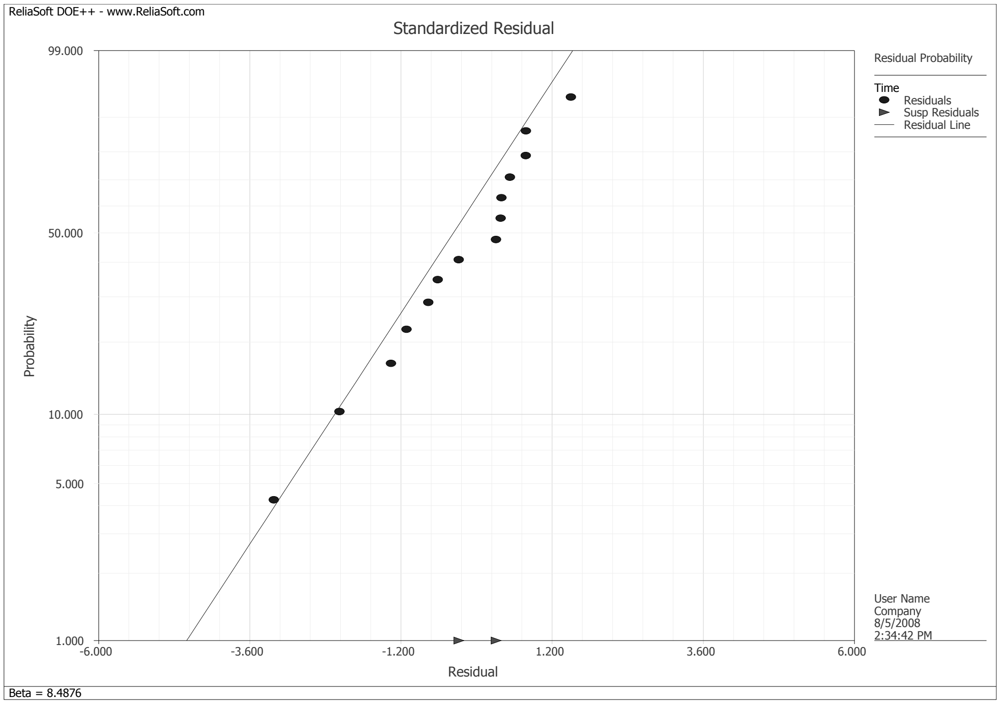
   Thus, based on the experimental results, the best settings to maximize the signal-to-noise ratio while remaining on the desired 2mm target are:

   - Speed: medium setting
   - Temperature: medium setting
   - Pressure: low setting
   - Order of metals: arrangement 2

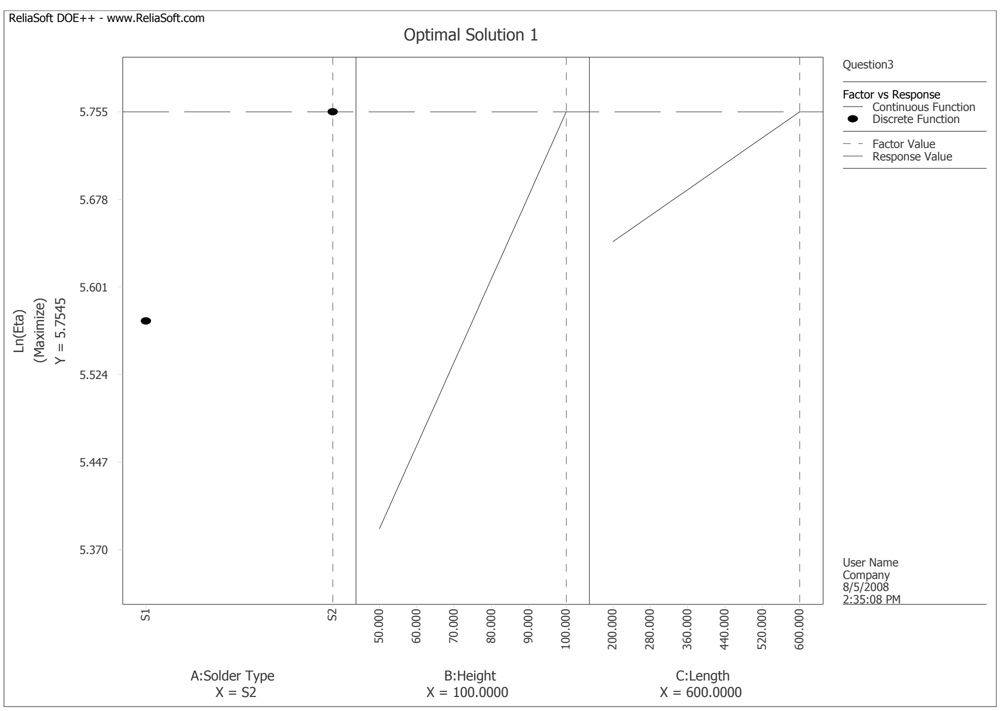
   Since the goal is both to maximize the signal-to-noise ratio and to keep the deposition thickness on target, trade-offs need to be made in the selection of factor settings.

## 6.3 Practice Question 3

3. Although the points do not track the line exactly, no significant deviations are found.




4. Height is the key factor, based on the information in the Analysis tab and/or the pareto chart. In addition, the interaction between solder type, height and length is significant.
6. Within the factor constraints (*i.e*. height between 50-100 microns, length between 200-600 microns), the optimal settings are found to be solder type = S2, height = 100, length = 600.



## 6.4 Practice Question 4

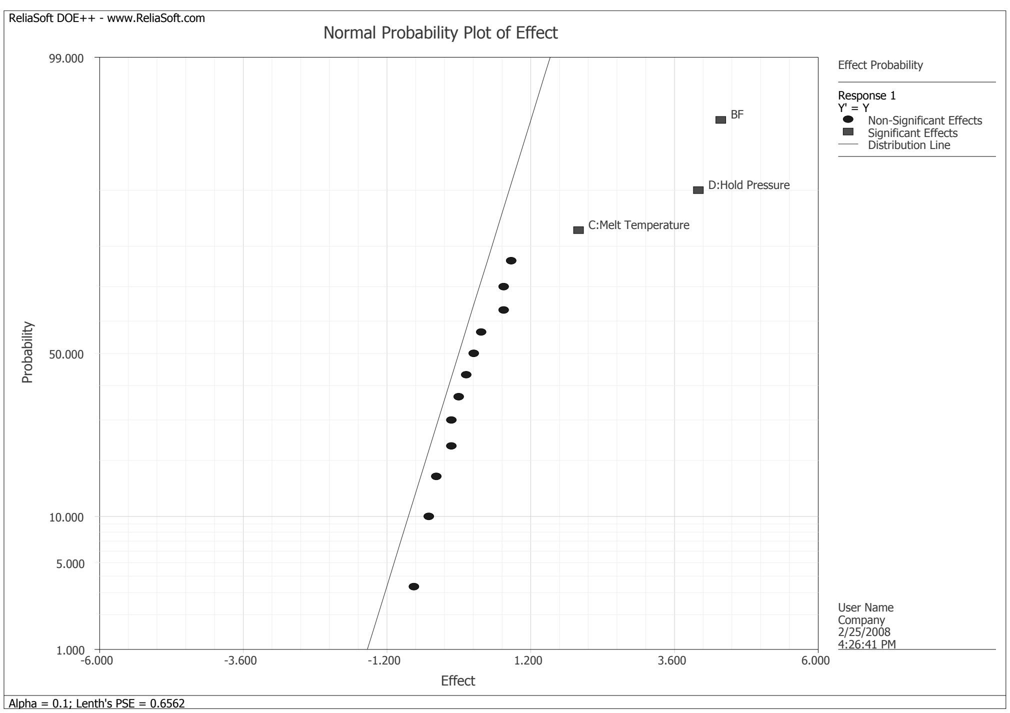
2. The alias structure is shown next.

| Initial Alias Structure: | | |
|---|---|---|
| | | |
| | Fraction Generators: | |
| | E = ABC | |
| | F = BCD | |
| | | |
| | [I] = I + ABCE + ADEF + BCDF | |
| | [A] = A + BCE + DEF + ABCDF | |
| | [B] = B + ACE + CDF + ABDEF | |
| | [C] = C + ABE + BDF + ACDEF | |
| | [D] = D + AEF + BCF + ABCDE | |
| | [E] = E + ABC + ADF + BCDEF | |
| | [F] = F + ADE + BCD + ABCEF | |
| | [AB] = AB + CE + ACDF + BDEF | |
| | [AC] = AC + BE + ABDF + CDEF | |
| | [AD] = AD + EF + ABCF + BCDE | |
| | [AE] = AE + BC + DF + ABCDEF | |
| | [AF] = AF + DE + ABCD + BCEF | |
| | [BD] = BD + CF + ABEF + ACDE | |
| | [BF] = BF + CD + ABDE + ACEF | |
| | [ABD] = ABD + ACF + BEF + CDE | |
| | [ABF] = ABF + ACD + BDE + CEF | |

All main effects are free from aliasing with main effects and second order interaction effects. However, the two-factor interactions may be aliased with each other.

If important two-factor interactions are found to be present, additional experimental trials should be conducted to separate the confounded effects.

3.




A normal probability plot of the unreplicated design shows the main effects of factors C and D and the interaction effect BF to be significant. From the alias structure it can be seen that the interaction effect BF is confounded with CD. Thus the actual source of this effect cannot be known on the basis of the present experiment. However since neither factor B nor F was found to be significant there is an indication that the observed effect is likely due to interaction CD. To confirm this, a follow-up $2^2$ experiment can be run involving only factors B and F. If those factors are found (or assumed) to be inactive, the fitted regression model for the fuel cone design is:

$$\hat{y} = 7.6875 + C + 2 \cdot D + 2.1875 \cdot CD$$

## 6.5 Practice Question 5

2. Yield:

$$\hat{y}_1 = 94.9 + 0.735x_1 + 1.53x_2 + 0.45x_1x_2 - 1.52x_1^2 - 2.08x_2^2$$

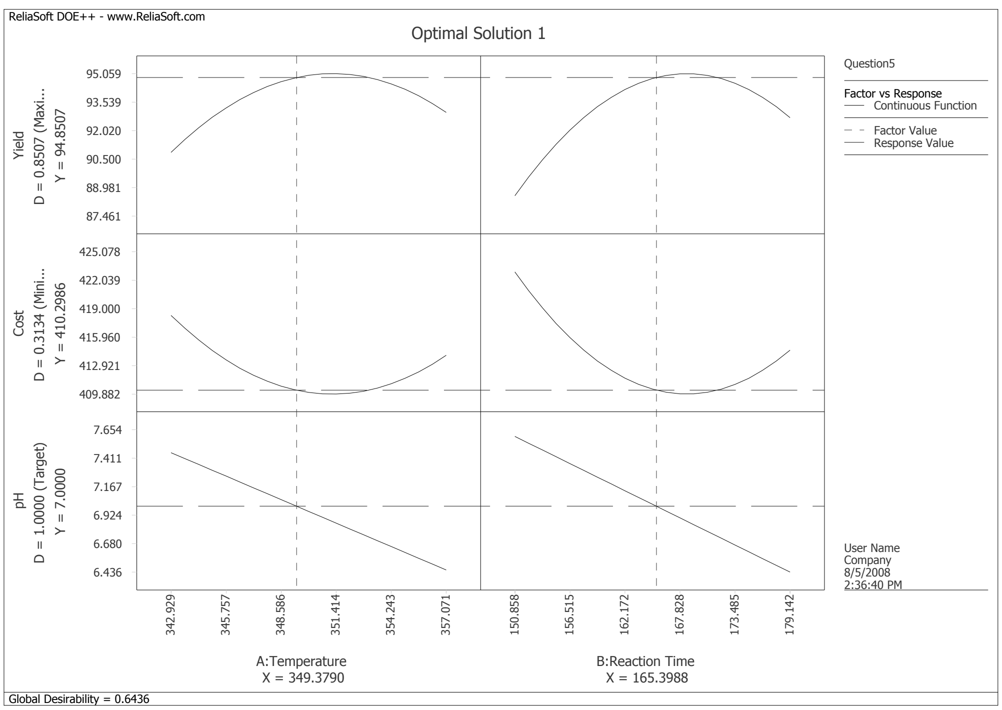
Cost:

$$\hat{y}_2 = 410.18 - 1.471x_1 - 3.06x_2 - 0.9x_1x_2 + 3.04x_1^2 + 4.17x_2^2$$

pH:

$$\hat{y}_1 = 6.97 - 0.355x_1 - 0.041x_2$$

3.




Optimum settings are found to be at a temperature of $x_1 = 349^oF$ and with a reaction time of $x_2 = 165$ minutes with an overall desirability of 0.6436. Note that the optimization solution shown above was found using a random seed of 1 (set on the Algorithm Settings page of the Optimization Settings window). If you use a different seed or no seed, your results may vary slightly.